Card Race

Année 2024 – 2025

Olivia OUADHI ALLARD ; Julian CESARI; Evan STENGER; Mathys DEROBERTMASURE; Stan FILLOU

Établissement(s) : Lycée Raynouard (Brignoles - Var)

Enseignant·e(s) : Mourau Nelly, Berger Antoine, Guicheteau Denis

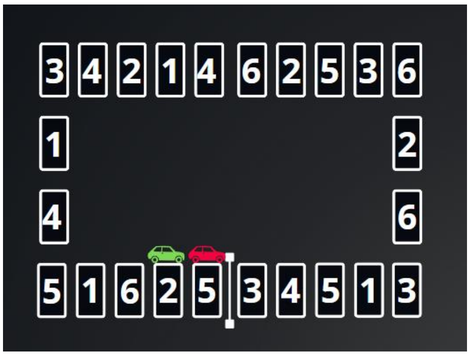
Chercheur·Chercheuse(s) : Thierry Champion laboratoire imath. Frédéric Havet - INRIA

1. Game rules

We will start by explaining the rules of the game.
We randomly place twenty-four cards numbered from one to six in a way that forms a circuit. There are therefore four ones, four twos, four threes, etc.

We randomly determine a finish line and place two cars on the two cards behind this line(1).

The cars move forward by the number indicated on the card they are on.



The question we want to answer is : what is the probability that the two cars arrive on the same cards after a lap(2).

2. Results

With 6 cards, the probability of arriving on the same card is 0,85.

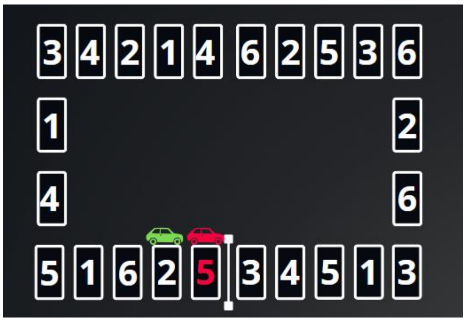
With 4 cards, the probability of arriving on the same card is 0,95.

3. Researches

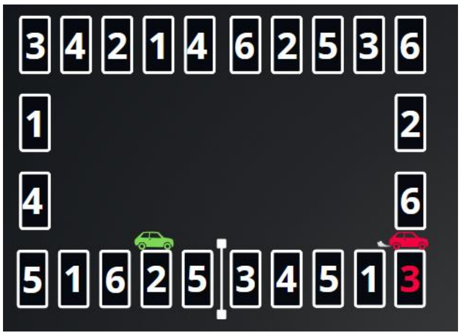
3.1. Presentation

We put two cars (one red and one green) on two cards just before the start line.

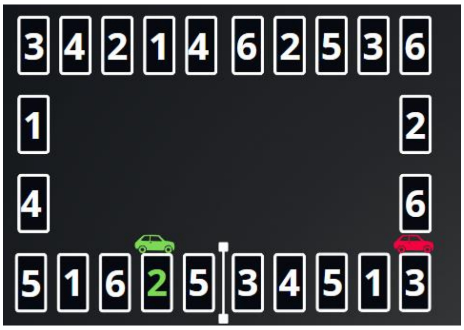
For the red car, it will move five cards forward because the number in the card is 5.



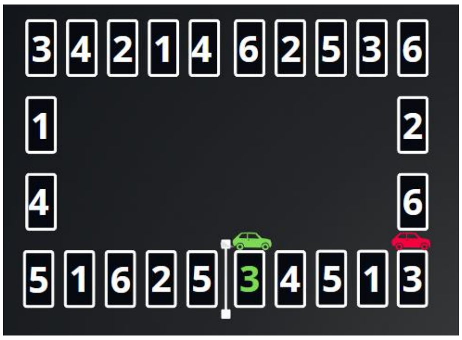
It then lands on the three(3).



The green car, on the other hand, moves two cards forward.



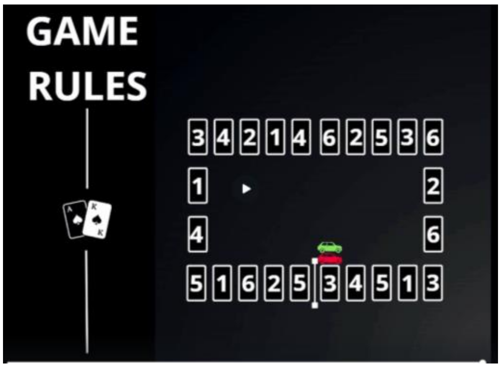
It lands on a three.



We repeat this action until both cars have completed the first lap.

As we observe, the red car moves to the six, while the green car moves to the one. The cars continue advancing until both have crossed the finish line. They then stop.

At the end, we notice that both cars land on the same card.



We can therefore ask ourselves whether this is a special case or a general rule.

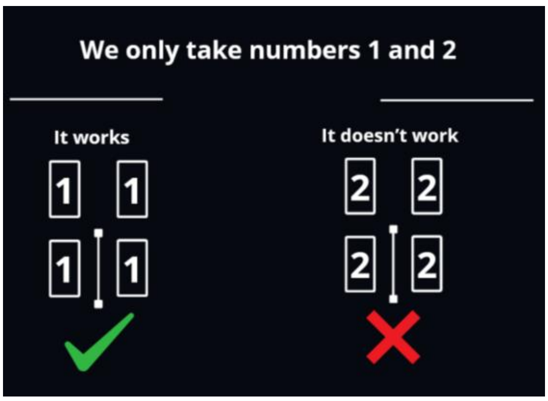
3.2. First steps in research

3.2.1. With only 4 cards with one :

We know that if we only have one, we will always land on the same card.

3.2.2. With only 4 cards with two :

However, with only twos, we never land on the same card because the cars will never be able to meet(4).

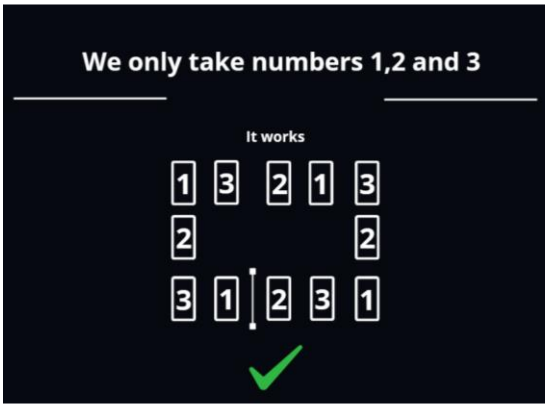


3.2.3. With one and two(5):

When we have only ones and twos, we systematically land on the same card because the ones regulate(6) the distance between the cards.

3.2.4. With only one, two, three:

We conjecture it is the same applies to cards ranging from one to three because we have not found a configuration that doesn't work.

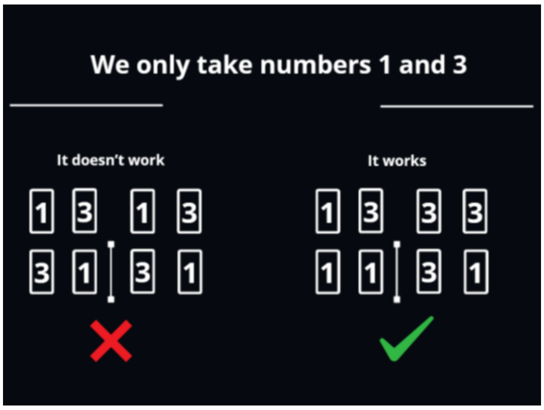


3.2.5. With only one and three:

However, with only ones and threes, things become more complicated.

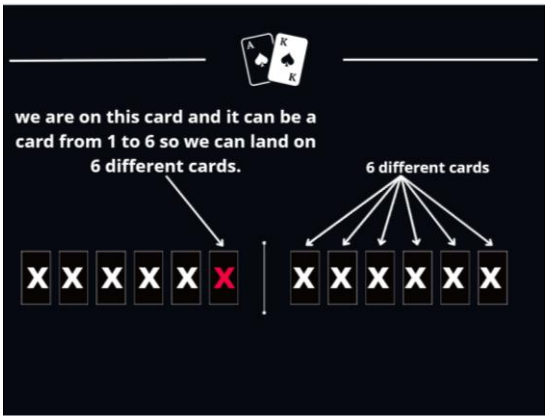
When we swap the ones and threes, we never land on the same card.

But when the ones and threes are placed randomly, there is a higher chance of landing on the same card because they regulate movement in a similar way to the twos.

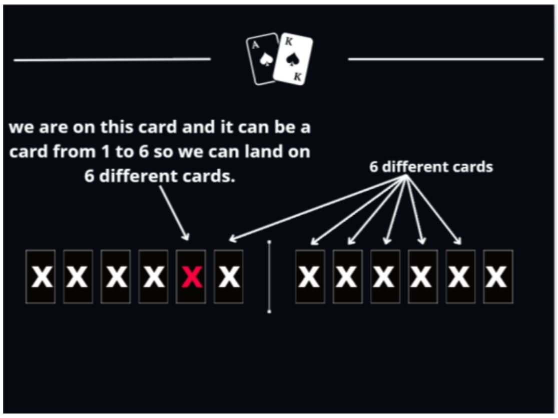


We also noticed that at the moment when the cars land on the same card during the lap, they will inevitably land on the same card at the end of the lap.

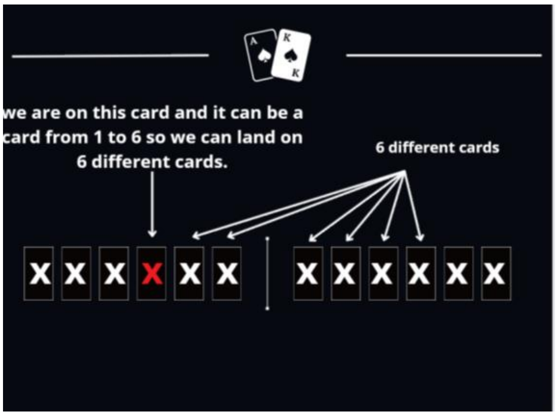
3.3. Another way of seeing things:



I tried to determine the probability of landing on a card. I realized that, since the highest card is a 6, you can land at most on the sixth card after the finish line.
So I wanted to find the probability of landing on each of the first 6 cards as a function of the card before the finish line. So for each of these 6 cards



I did the same for the second card before the finish line, so now we can fall on all 6 cards after this one. Since we can now fall on the first card before the finish line, we must again calculate the probability for the first card before the finish line.



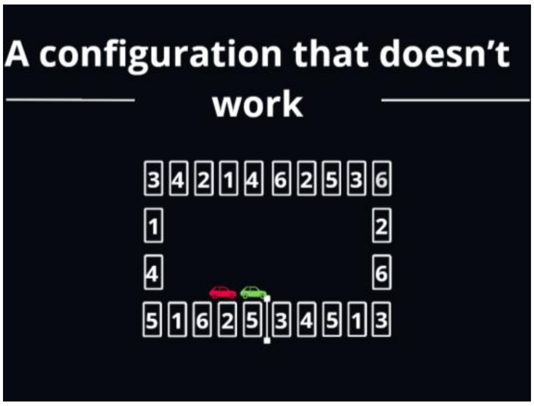
And the same goes for the third card before the finish line. And now we can fall on the second or first card before the finish line, so we can either fall directly on the first card and recalculate the probability with the first card, or fall on the second card and repeat what I explained on the previous slide.
But i didn't go to far on that way.

3.4. A configuration that doesn't work with twenty-four cards

We have seen that in approximately fifteen percent of configurations, the cars do not land on the same card at the end of the first lap.

This is the case with this particular sequence of cards.

We can clearly see that they do not land on the same card(8).



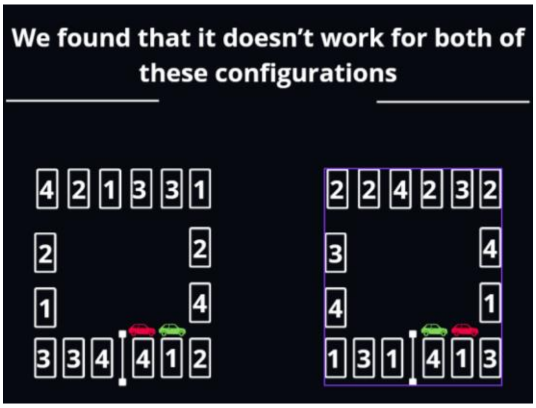
3.5. A configuration that doesn't work with sixteen cards

We will now work with cards numbered from one to four, meaning we remove all the fours and fives from our set of cards.

We now have sixteen cards to play with.

We then selected two configurations that do not work from the five percent of cases where this occurs.

We can see that in both selected configurations, the cars do not land on the same card.



We found the ratio of configuration that worked with a program, because we failed to calculate it.

4. Program(7)

```
# Initialisation
num_cartes = 16
cartes = [1, 2, 3, 4] * 4
random.shuffle(cartes)  # I
```

```
# Initialisation
num_cartes = 24
cartes = [1, 2, 3, 4, 5, 6] * 4
random.shuffle(cartes)  # Mélange
```

In this part of the program we create a list containing numbers from 1 to 6. each number will be repeated 4 times. Then we randomly mix the list. We can also take cards from 1 to 4 .

```
# Fonction pour simuler un parcours
def simulate_parcours(cartes, num_simulations=1):
    # Compteur pour chaque carte
    comptage = np.zeros(num_cartes)

    for i in range(num_simulations):
        position = 0  # Commence avant la ligne de départ
        for position in range(4):
            visited = set()  # Pour éviter les boucles infinies
            while position not in visited:
                visited.add(position)
                # Avancer selon la valeur de la carte
                avance = cartes[position]
                position = (position + avance) # Le circuit est cyclique
                if position >= num_cartes:  # Si on dépasse la ligne de dépar
                    position=position%num_cartes
                    break
            # Comptabiliser la carte où on s'est arrêté
            comptage[position] += 1
```

We use this function to simulate 4 car's trips. We place each car on the 4 first cards and we save the final position in a new list (we count how many times it arrives on the same card)

```
    # Calcul des probabilités d'atterrir sur chaque carte
    probabilites = comptage /(4* num_simulations)
    return probabilites
```

Then we calculate the probability to land on each card. If there is a card wich has a probability of 1, this means that all the cars land on it.

```
for i in range(nbfois):
    random.shuffle(cartes)
    probabilites = simulate_parcours(cartes)
    if 1 in probabilites :
        cpt+=1
```

We do it a lot of time and count the ratio of how many simulations give a good result.

With the sixteen cards with the value 1 to 4, we found around 95% of chance that the 4 cars stopped in the same card

```
Powered by trinket
0.95181
```

With the twenty-four cards with the value 1 to 6, we found around 85%.

```
Powered by trinket
0.84899
```

(9)

5. Conclusion

To conclude, with the Python program we found a probability of 0,85 for a deck of 24 cards with the cards 1 to 6. for a deck of 16 cards with the cards 1 to 4 we found a probability of 0,95.(10)

We didn't manage to prove any of these probabilities, and this is the work we have to do now.

Notes d'édition

(1) The 2 cars are one behind the other.

(2) what is the probability that the two cars arrive on the same cards after a lap and *after crossing the finish line*

(3) « On the three » means « on the card which number is 3.

(4) In fact, we can say : « However, with only twos, the cars will never land on the same card because their way is cyclic ».

(5) Here the study is with only 8 cards with one and two

(6) As soon as there are two consecutively numbered cards with 1, the cars follow the "same path."

(7) With 16 cards: https://trinket.io/python3/a02699c8dc19. With 24 cards: https://trinket.io/python3/008928ed7d8c

(8) The sequence 2/3/1/3/1/2/2/3/1/2/3/1 answers the conjecture in the negative

(9) The probabilities are derived from 100,000 simulations; at least 1 million simulations should be considered.

(10) In the case of 16 cards with values from 1 to 4, there are 63,063,000 possible configurations; systematic processing would yield the exact probability.