Pizza Time

Math en Jeans - Year 2017-2018

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Abstract

Our team will present various problems of geometry and algebra related to a common subject. It looked pretty easy in the beginning, starting with suggestive drawings, but the Maths proved to be difficult. The name of the topic is *Pizza Time* and the questions focus on situations commonly encountered. First question is about finding the person who ate more pizza when it is cut in a specific way, and to solve it we used helpful constructions. Another question deals with the number of toppings for a restaurant that claims to have 1001 combinations of pizza toppings. We used some combinatorial formulas and obtained an interesting result for the restaurant. There were also questions related to the diameter of the pizza or to specific ways of cutting the pizza, for which we finally came up with solutions. We tried to do our best in making the presentation a really attractive one, such that everybody will enjoy, and we have used our creativity and attention to solve these problems.

The problem

1. A pizza is sliced by doing two perpendicular cuts, of lengths *x+6* and *16* cm (see Figure 1). Alin chooses the slices marked with blue and Cristian the ones with red. Who ate more pizza and how much ?

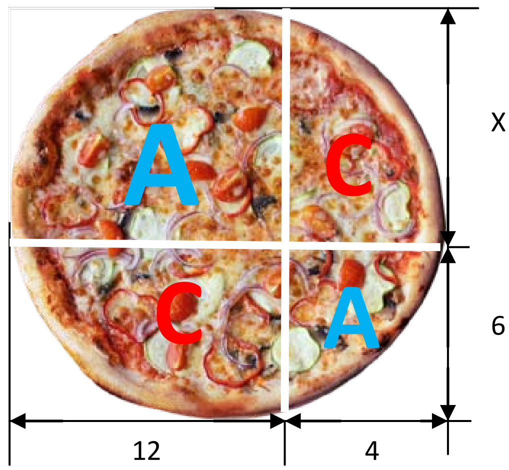


Fig.1

2. Tim slices the pizza in an unconventional way (Fig.2). Instead of cutting it radial, he decides to cut it in four strips of equal width, both vertically and horizontally. The area of the smallest piece of pizza is$12\pi + 36(1-\sqrt{3})cm^2$. Calculate the diameter of the pizza.

Figure. 2

3. What is the maxim number of pieces that you can get if you cut a pizza with 12 straight cuts?

4. Find a way to cut a circular pizza into 12 congruent pieces so that at least a piece of the 12 does not contain a margin.

5. A restaurant offers pizza with*"1001 different combination of toppings"*. To order a pizza, the customer can combine, on request, the types of favorite toppings by choosing the number of portions of each type. How many toppings does that restaurant offer if a desired topping can be chosen at least once?

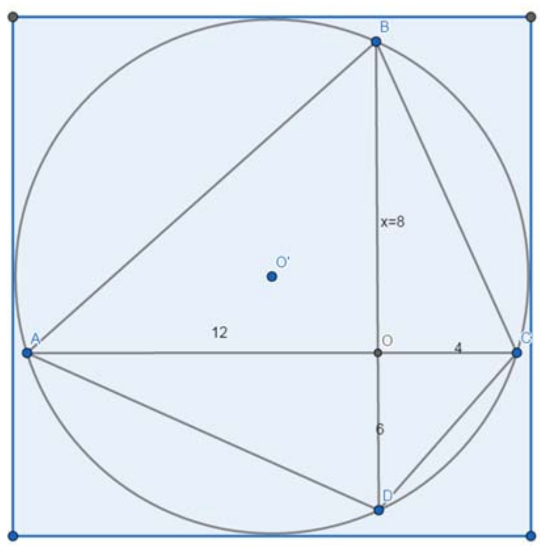
First of all, after cutting the pizza by two perpendicular cuts, four rectangular triangles will be formed, all of which have their right angle at the intersection point of the straight lines. We compared two of the triangles and the angle-angle case turned out to be similar. Due to this, the sides of the two triangles will be proportional. Replacing this report results that $x = 8cm$.

We decided to construct the symmetries of the two straight lines and after that we found that each piece would be reduced one by one, and in the end it will remain only the middle part, a rectangle of dimensions 8 and 2 that is part of Alin's piece.

In conclusion, Alinate with 16 cm^2 more than Cristian.

Solution of the problem

1. *If A,B,C,D ∈C (O', r); AC∩ BD= {O}*



We observe that the angles $\widehat{AOD}$and $\widehat{COD}$ are congruent, as the angles are 90° ($AC \perp BD, AC \cap BD = \{O\}$). At the same time, the angles $\widehat{DAO}$and$\widehat{CBO}$ are congruent because "in a quadrilateral quoted in the circle, the angles formed by the diagonal with the opposite sides are congruent"

We will compare *ΔAOC* and *ΔBOD*.

ΔAOC, ΔBOD : $\widehat{AOC} \equiv \widehat{BOD}$, $\widehat{DAO} \equiv \widehat{CBO}$

And by the angle-angle case it results that the two triangles are similar.

$$\Delta AOC \sim \Delta BOC$$

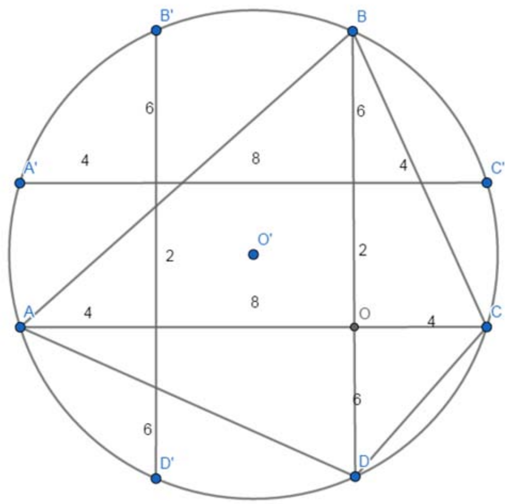
Hence, the corresponding sides of the triangles are proportional.

$$\frac{OC}{OD} = \frac{OB}{OA} = \frac{BC}{AD}$$

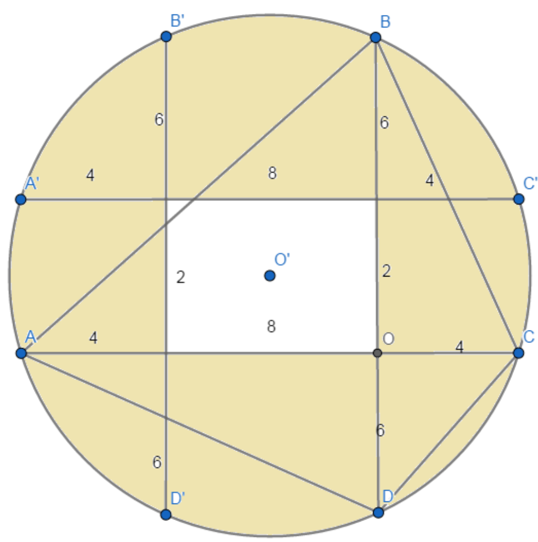
By replacing the resulting report, it will result in the following relationship:

$$\frac{OC}{OD} = \frac{OB}{OA} \Rightarrow \frac{4}{6} = \frac{x}{12} \Rightarrow x = \frac{12 \cdot 4}{6} \Rightarrow x = 8$$

We will construct the symmetric lines of *AC* and *BD*, along the diameters of the circle. And if we overlap the parts eaten by each boy they will be reduced one by one. First, reduce the portion containing the *A'B'* arc to the *BC'* arc and then the portions with the *AD'*and*CD* arcs. Then the *BB*' arc with the arc *DD*' and last but not least the *AA'* and *CC'* sections



Finally, the drawing will look like below, and the remaining portion will be a rectangle of *L = 8, l = 2.*

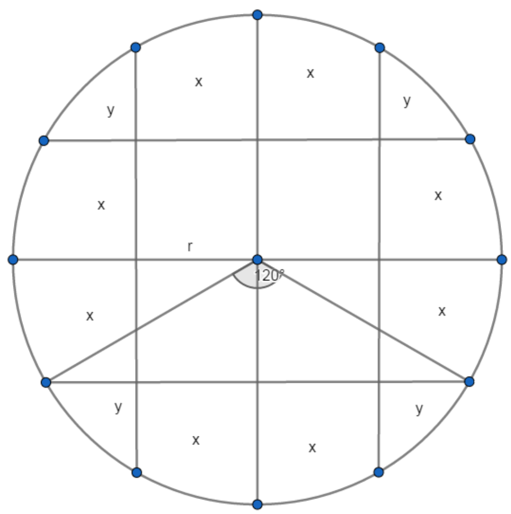


The resulting rectangle area will be the difference of pizza between the two children and this belongs to Alin's portion.

$$A_p = L \cdot l = 8 \cdot 2 = 16cm^2$$

In conclusion, Alin ate more pizza with 16 cm^2.

2. This question is based on circle formulas. We decided to write the congruent parts in the same letters to write the areas in 2 ways. We will make a system of equations between the disk area and the area of a circle sector having the center angle of 120 degrees.
   We denote by
   y- the smallest piece of pizza, r- the radius of pizza, x- each piece next to the one marched with y



The disk area will be equal to:

$$A_{disk} = \pi r^2 = 8x + 4y + r^2$$

The area of the sector is:

$$A_{sector} = \frac{1}{3}\pi r^2 = 2x + 2y + \frac{\sqrt{3}}{2} \cdot \frac{r^2}{2}$$

We will get a system of equations, we will multiply the 2nd relation with -4, and after we add them, we will replace y.

$$\begin{cases} \pi r^2 = 8x + 4y + r^2 \\ \frac{1}{3}\pi r^2 = 2x + 2y + \frac{\sqrt{3}}{2} \cdot \frac{r^2}{2} \end{cases}$$

$$\begin{cases} -8x - 4y = r^2 - \pi r^2 \\ -2x - 2y = \frac{r^2\sqrt{3}}{4} - \frac{\pi r^2}{3} /\cdot\ (-4) \end{cases}$$

$$\begin{cases} -8x - 4y = r^2 - \pi r^2 \\ 8x + 8y = r^2\sqrt{3} + \frac{4\pi r^2}{3} \end{cases} |+$$

$$\boldsymbol{y} = \frac{r^2}{4} + \frac{\pi r^2}{12} - \frac{r^2\sqrt{3}}{4}$$

$$(=)\boldsymbol{y}=\frac{\boldsymbol{r^2-r^2\sqrt{3}}}{\boldsymbol{4}}+\frac{\boldsymbol{\pi r^2}}{\boldsymbol{12}},\quad \boldsymbol{y=12\pi+36\left(1-\sqrt{3}\right)}$$

$$\Rightarrow 12\pi+36-36\sqrt{3}=\frac{r^2-r^2\sqrt{3}}{4}+\frac{\pi r^2}{12}$$

$$(=)\frac{r^2-r^2\sqrt{3}}{4}+\frac{\pi r^2}{12}-12\pi-36+36\sqrt{3}=0$$

$$(=)\frac{3\left(r^2-r^2\sqrt{3}\right)+\pi r^2-144\pi-432+432\sqrt{3}}{12}=0$$

$$(=)3r^2-3\sqrt{3}r^2+\pi r^2-144\pi-432+432\sqrt{3}=0$$

$$(=)3r^2-3\sqrt{3}r^2+\pi r^2=144\pi+432-432\sqrt{3}$$

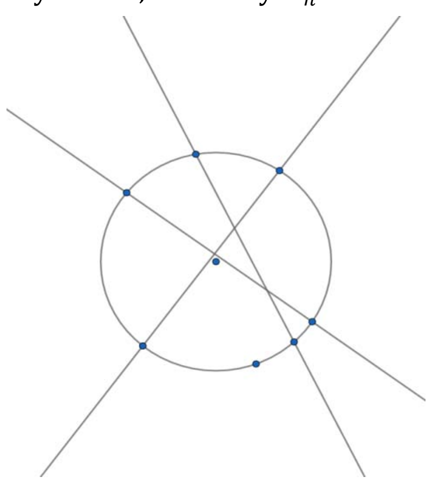
$$\Rightarrow r^2=\frac{144\pi+432-432\sqrt{3}}{3-3\sqrt{3}+\pi}$$

$$(=)r^2=\frac{144(\pi+3-3\sqrt{3})}{3-3\sqrt{3}+\pi}$$

$$\Rightarrow r^2=144\Rightarrow r=12\ cm$$

The pizza diameter is equal to *2r* and it is *24 cm.*

3. For any $n\in\mathbb{N}^*$, denote by d_n = the number of slices witch we get after n cuts



If we do three random cuts, then the maximum number of pieces is 7.

We are looking for a recurrence relation for d_n.

nr. cuts	*0*	*1*	*2*	*3*	*4*	*5*	*…*	*n*
nr. pieces	*1*	*2*	*4*	*7*	*11*	*16*	*…*	*?*

$$d_{n+1}=d_n+n+1, n\geq 0 \quad \underline{(1)}$$

$$\mathrm{d}_{\mathrm{n}+1} - \mathrm{d}_0 = \sum_{\mathrm{k}=10}^{\mathrm{n}} (\mathrm{k}+1) = \sum_{\mathrm{k}=1}^{\mathrm{n}+1} \mathrm{k} = \frac{(n+1)(n+2)}{2}$$

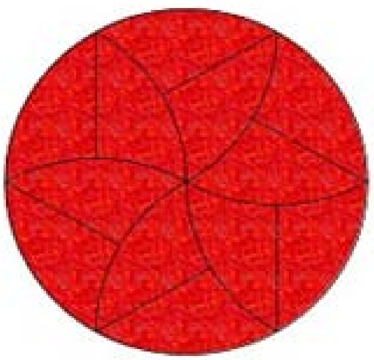
$$d_{n+1} = 1 + \frac{(n+1)(n+2)}{2}$$

If pizza is sliced using 12 cuts, that means *n = 11*, then.

$$d_{12} = 1 + \frac{12 \cdot 13}{2} = 79$$

In conclusion, a pizza cut through 12 straight cuts will result in a maximum of 79 pieces.

4.



Joel Haddley and Stephen Worsley, from the University of Liverpool came up with a previous method for cutting the perfect slice, known as **monohedral disk tiling**, which results in 12 identical slices.
This process begins by cutting the pie into six curved three side shapes cross the pie.

If done right, it will look similar to a star shape coming out of the center.

Then divide those shapes in two, resulting in an inside group, no crust, and an outside group, with crust.

Source: Daily Mail

5. *In mathematics, a **combination** is a selection of items from a collection, such that (unlike permutations) the order of selection does not matter. Combinations refer to the combination of n things taken k at a time without repetition. To refer to combinations in which repetition is allowed, the terms k-selection, k-multiset, or k-combination with repetition are often used. (**Source: Wikipedia)***

We use the formula for combinations on *n* objects taken by *k* without or with repetitions.

In the first case, we assume that a topping cannot be repeated on the same pizza (the order does not matter). The number of possibilities is

$$\binom{n}{k} = \boldsymbol{C_n^k} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{k})! \cdot \boldsymbol{k}!}, \quad 0 \le k \le n$$

For 1001 combinations, we will factorize1001 as:

$$1001 = 7 \cdot 11 \cdot 13 = \frac{11 \cdot 12 \cdot 13 \cdot 14}{24} = \frac{11 \cdot 12 \cdot 13 \cdot 14}{4!} = \frac{14!}{10! \cdot 4!} = \binom{14}{4} \quad (2)$$

It follows that we will have 14 different toppings, and up to 4 toppings can be selected on each pizza, but a topping can be chosen exactly once.

In the second case, we assume that a topping can be chosen multiple times. Then, the number of possibilities is

$$\binom{14}{4} = \binom{n+k-1}{k-1} \quad (3)$$

In the end, we will have 10 different toppings, which can be chosen and 5 different ways with repetition.

Conclusion

In conclusion, all the questions were solved and explained in our article. They can also be useful in everyday life, not only to find mathematical relationships, but also to help us develop our thinking. We have found that approximations can change a radical result, and if a pizza restaurant would do that it would have much bigger loss. Also, a simple idea can give you the answer right away without any heavy and complicated calculations. We believe that our solution is very interesting and we have used our knowledge and creativity to the maximum to have a final answer to each problem.

References:

Daily Mail- *Mathematicians reveal the perfect way to cut pizza: 'Spiky' shapes allow unlimited number of equal slices*

Wikipedia- *Combination*

Clubul Matematicienilor - *Matematica Evaluare Nationala*

Notes d'édition

(1) Cette égalité doit être justifiée : En effectuant une coupe supplémentaire après n coupes, cette nouvelle coupe rencontre au maximum ces n coupes, ce qui crée au maximum n+1 nouvelles régions.

(2) Il faudrait vérifier qu'il n'y a pas d'autre solution possible.

(3) Formule à justifier : nombre de combinaisons lorsque les répétitions sont autorisées.