Maths for the best match

Year 2023 – 2024

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Keywords: probability, IQ, choice, optimal strategy

Abstract

Our research deals with finding the probability and the optimal strategy that a chief of a tribe would find the girl with the highest IQ, in order to marry her. We propose to the chief the "k-strategy" to achieve his goal. We eliminate some of the possible candidates, regardless of their IQ and then choose the best option higher than the rejected ones. If he does not choose the woman with the highest IQ, he will not get married this year.

The statement of the problem

The chief of a certain tribe decided that he is ready to get married. In his opinion, the best match for him would be the smartest lady from his tribe. However, the tribal council decides that he must also show some sign of wisdom, at least by proving that he is a good strategist.

Each of the 73 single ladies in this tribe was given a certain IQ test, which was approved a priori by the council. All the IQ scores received by the ladies were different from each other. Each score was written on a piece of paper and then placed inside an opaque ball. All the 73 balls were placed and shuffled inside an urn. The chief's goal is to find the ball containing the highest IQ score, that corresponds to his best match.

The tribal chief randomly (uniformly) picks a ball from the urn and sees the score written inside the ball. He must decide whether to keep this score or reject it. If he decides to keep it and this number is the highest score in that urn, then he will get to marry the smartest lady. Otherwise, if the score is not the largest, he will remain unmarried this year.

If the chief rejects the score written inside the picked ball, then this score is removed from the urn. He randomly picks a new ball from the urn, and the previous procedure is repeated. He can either accept the score written inside this ball or refuse it. The procedure of picking balls continues like this until either he accepts one score or else the balls are exhausted. If there is only one ball left in the urn, the chief has to accept the score written inside of it.

The chief does not know the scores, nor their distribution inside the urn. At each moment, he only knows how many balls remain in the urn and the scores that he rejected. Moreover, he cannot change his mind and go back to choose a score that he has already rejected.

How should the chief of the tribe proceed to have the best chance of getting married this year? What is the probability of success with this strategy? If the number of ladies in the tribe is much larger that 73, is there any reasonable hope of finding the smartest wife?

1. Elements of probability (1)

Probability is a branch of mathematics concerning events and numerical descriptions of how likely they are to occur. The word *probability* has several meanings in ordinary conversation, which are particularly important for the development and applications of the mathematical theory of probability. One is the interpretation of probabilities as relative frequencies, for which simple games involving coins, cards, dice, and roulette wheels provide examples. In loose terms, we say that the probability of something happening is 0.25, if, when the experiment is repeated often under the same conditions, the stated result occurs 25% of the time.

A *random experiment* is a process by which an observation is made; an observation is referred to as an *outcome*. The outcome of a random experiment cannot be determined before it occurs, but it may be any one of several possible outcomes. The outcomes are considered to be determined by chance. In our paper, we will confine our discussion to cases where there are a finite number of equally likely outcomes. For example, if a coin is tossed, there are two equally likely outcomes: heads (H) or tails (T). If a die is tossed, there are six equally likely outcomes: $1, 2, 3, 4, 5, 6$.

The *sample space* (denoted here by S) is the set of all possible outcomes of a random experiment.

An *event* is (or random event) a set of outcomes of an experiment or a subset of the sample space. A *simple event* (or elementary event) is an event that cannot be decomposed. Events are usually denoted by capital letters, such as $A, B, C, \ldots$

The *probability* of an event numerically quantifies the chances of that event to occur. We shall denote the probability of event A as $P(A)$. In the classical theory of probability (finite sample space with equally probable simple events), the probability of an event A is defined as ratio between the number of favourable cases to A to the number of cases in the sample space. We write this mathematically as:

$$P(A)=\frac{\textit{number of elements in } A}{\textit{number of elements in } S}=\frac{\mathrm{card}(A)}{\mathrm{card}(S)}.$$

Probabilities will always be between (and including) 0 and 1. A probability of 0 means that the event is impossible. A probability of 1 means an event is guaranteed to happen (2). A probability close to 0 means the event is "not likely" and a probability close to 1 means the event is "highly likely" to occur.

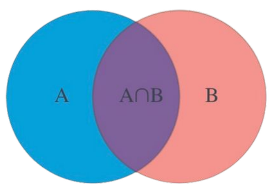
If A is any event, the event that A does not occur is called the *complement* of A, denoted by $\overline{A}$. The complementary event of is made of all the outcomes that are not associated with the event but are in the sample space Furthermore,

$$P(\overline{A})=1-P(A).$$

If A and B are two events defined on the sample space S; the *union of A and B* (written as $A\cup B$) is the event that either A or B occurs or both occur.

If A and B are two events defined on the sample space S; the *intersection of A and B* (written as $A\cap B$) is the event that both A and B occur simultaneously. We have that

$$P(A\cup B)=P(A)+P(B)-P(A\cap B).$$



We say that the events A and B are *mutually exclusive events* (written as $A\cap B=\varnothing$) if the two events have no outcomes in common. If A and B are mutually exclusive events, then

$$P(A\cup B)=P(A)+P(B).$$

The existence or absence of an event can provide clues about other events. In general, an event is deemed dependent if it provides information about another event. An event is deemed independent if it offers no information about other events. In other words, a dependent event can only occur if another event occurs first. We shall write this mathematically.

If A and B are events of S ($P(B)\neq 0$), and the occurrence of event A is dependent on the occurrence of event B (written $A|B$), then A *is conditional on* B and the probability of A given B is

$$P(A|B)=\frac{P(A\cap B)}{P(B)}.$$

Similarly, if $P(A)\neq 0$) and the occurrence of event B is dependent on the occurrence of event A (written $B|A$), then B *is conditional on* A and the probability of B given A is:

$$P(B|A)=\frac{P(A\cap B)}{P(A)}.$$

From the previous two relations we conclude that

$$P(A\cap B)=P(B)P(A|B)=P(A)P(B|A).$$

Two events, A and B, are *independent* if the occurrence of A does not affect the probability of the occurrence of B; otherwise the events are said to be *dependent*. If A and B are two independent events, then $P(A|B)=P(A)$ and $P(B|A)=P(B)$, and thus

$$P(A\cap B)=P(A)P(B).$$

If the event A may depend on more events, say B_1, B_2,..., B_k, that form a *partition of the sample space* (that is, the union of them is the sample space and they are mutually exclusive events), then we have (*the law of total probability formula*):

$$P(A)=P(A|B_1)\cdot P(B_1)+P(A|B_2)\cdot P(B_2)+...+P(A|B_k)\cdot P(B_k)=\sum_{i=1}^{k} P(A|B_i)P(B_i).$$

If the occurrence of event A is dependent on the occurrence of event B, then the occurrence of event A is also dependent on the occurrence of event $\overline{B}$. Then, B and $\overline{B}$ form a partition of the sample space. In this particular case, the law of total probability formula becomes:

$$P(A)=P(B)\cdot P(A|B)+P(\overline{B})\cdot P(A|\overline{B}).$$

2. The solution of the problem

For generality, let us consider an urn that has n indistinguishable balls. In our problem, $n=73$. Also, let us denote by B_n the ball containing the highest number inside an urn having n balls. We shall call it *the winning ball*.

Since we do not know anything about the distribution of the numbers written inside the balls, a reasonable strategy is to wait until a certain number of balls (say, k) are withdrawn, without accepting any of them, and then accept the ball containing the highest number that appears thereafter, if that exists.

Obviously, the number k is an integer between 0 and $n-1$. If $k=0$, then we accept the first ball drawn, while if $k=n-1$, then we accept the last ball from the urn. We shall call this strategy as the "k-strategy".

Our aim is to find the probability of success using the "k-strategy". We call a *success* when the tribal chief accepts the ball with the highest number inside it.

If n is small, one can find the best "k-strategy" by hand. We start by considering simple cases $n=2, n=3, n=4, \ldots$, to see whether there is a pattern.

The case n = 2

There are $2!=2$ possible permutations of the 2 balls corresponding to the 2 possible extraction configurations of the 2 balls from the urn. As we want all the possible extraction configurations to be equally probable, we assume that the extraction continues until all the balls are extracted from the urn, even when the winning ball is accepted (3). If this assumption is not considered, then one cannot calculate the classical probability of a success. In the extraction configurations below, we shall mark with red the balls that were rejected by default and with green the winning ball.

- If the chief accepts the first extracted ball, then there are no balls rejected by default. We calculate the probability of winning with the "0-strategy". Out of the total 2 extraction configurations, only 1 of them has the ball containing the highest number on the first position. The favourable configuration is $(2,1)$.

Thus, the probability of success with the "0-strategy" is $P_{0,2}=\frac{1!}{2!}=0.5$.

- Suppose that the chief rejects the first extracted ball, and then accepts the ball that contains a number higher than all previously extracted numbers. The favourable winning configuration for the "1-strategy" is $(1,2)$.

k	$k=0$	$k=1$
$P_{k,2}$	0.50	0.50

Then the probability of winning with the "*1*-strategy" is $P_{1,2}=\frac{1}{2}=0.5$.

Summarizing, the probabilities of success with the "k-strategy" (with 2 balls) are:

We see that either of "0-strategy" and "1-strategy" is optimal for the case of 2 balls.

The case n = 3

There are $3!=6$ possible permutations of the 3 balls corresponding to 6 possible extraction configurations of the 3 balls from the urn. As we want all the possible extraction configurations to be equally probable, we assume that the extraction continues until all the balls are extracted from the urn, even when the winning ball is accepted. If this assumption is not considered, then one cannot calculate the classical probability of a success. In the extraction configurations below, we shall mark with red the balls that were rejected by default and with green the winning ball.

- If the chief accepts the first extracted ball, then there are no balls rejected by default. We calculate the probability of winning with the "0-strategy". Out of the total $3!$ extraction configurations, only $2!$ of them have the ball containing the highest number on the first position. The favourable configurations are:

$$(3,1,2),\quad (3,2,1)$$

Thus, the probability of success with the "0-strategy" is $P_{0,3}=\frac{2!}{3!}=0.33$.

- Suppose that the chief rejects the first extracted ball, and then accepts the ball that contains a number higher than all previously extracted numbers. The favourable winning configurations for the "1-strategy" are as follows:

$$(2,1,3),\ (1,3,2),\ (2,3,1)$$

Then the probability of winning with the "1-strategy" is $P_{1,3}=\frac{3}{6}=0.50$.

- Suppose that the chief rejects the first two extracted balls, and then accepts the ball that contains a number higher than all previously extracted numbers. The favourable winning configurations for the "2-strategy" are as follows: $(1,2,3)$, $(2,1,3)$.

Then the probability of winning with the "2-strategy" is $P_{2,3}=\frac{2}{6}=0.33$.

Summarizing, the probabilities of success with the "k-strategy" (with 3 balls) are:

k	$k=0$	$k=1$	$k=2$
$P_{k,3}$	0.33	0.50	0.33

We conclude that the "1-strategy" is optimal for the case of 3 balls.

The case n = 4

As all the four numbers are different from each other, the ranks of the numbers written inside the balls are 1, 2, 3 and 4. There are $4!=24$ possible permutations of the 4 balls corresponding to 24 possible extraction configurations of the 4 balls from the urn. As we want all the possible extraction configurations to be equally probable, we assume that the extraction continues until all the balls are extracted from the urn, even when the winning ball was accepted. If this assumption is not considered, then one cannot calculate the classical probability of a success. In the extraction configurations below, we shall mark with red the balls that were rejected by default and with green the winning ball.

- If the chief accepts the first extracted ball, then there are no balls rejected by default. We calculate the probability of winning with the "*0*-strategy". Out of the total $4!$ extraction configurations, only 3! of them have the ball containing the highest number on the first position. The favourable configurations are:

$$(4,1,2,3),(4,1,3,2),(4,2,1,3),(4,2,3),(4,3,1,2),(4,3,2,1).$$

Thus, the probability of success with the "0-strategy" is

$$P_{0,4}=\frac{3!}{4!}=0.25=P(B_4=1).$$

- Suppose that the chief rejects the first extracted ball, and then accepts the ball that contains a number higher than all previously extracted numbers. The favourable winning configurations for the "1-strategy" are as follows:

$$(1,4,2,3),(1,4,3,2),(2,4,2,3),(2,4,3,2),(3,4,1,2),(3,4,2,1),$$
$$(2,1,4,3),(3,1,4,2),(3,2,4,1),(3,1,2,4),(3,2,1,4),$$

Then the probability of winning with the "1-strategy" is $P_{1,4}=\frac{11}{24}=0.4583$

- Suppose that the chief rejects the first two extracted balls, and then accepts the ball that contains a number higher than all previously extracted numbers. The favourable winning configurations for the "2-strategy" are as follows:

$$(1,2,4,3),(2,1,4,3),(1,3,4,2),(3,1,4,2),(2,3,4,1),$$
$$(3,2,4,1),(1,3,2,4),(3,1,2,4),(2,3,1,4),(3,2,1,4).$$

Then the probability of winning with the "*2*-strategy" is $P_{2,4}=\frac{10}{24}=0.4167$.

- Suppose that the chief rejects the first three extracted balls, and then must accept the fourth (last). The favourable winning configurations for the "3-strategy" are as follows:

$$(1,2,3,4),(2,1,3,4),(1,3,2,4),(3,1,2,4),(2,3,1,4),(3,2,1,4).$$

Then the probability of winning with the "3-strategy" is $P_{3,4}=\frac{6}{24}=0.2500$.

Summarizing, the probabilities of success with the "k-strategy" $(k\in\{0,1,2,3\})$with 4 balls are:

k	$k=0$	$k=1$	$k=2$	$k=3$
$P_{k,4}$	0.2500	0.4583	0.4167	0.2500

We see that the "1-strategy" is optimal for the case of 4 balls.

The case $n \geq 5$

However, for n large we cannot count all the probabilities in this way, as the calculation becomes cumbersome.

Let us denote by $W_{k,n}$ the event of success (win) when using the "k-strategy" for an urn containing $n \geq 2$ balls.

Before starting to pick up any ball, the ball containing the highest number inside could be any of the n balls. As the tribal chief chooses randomly the balls from the urn, in an uniform manner, each of the n balls has the same chance of being drawn. The winning ball can be any of the balls in the urn, with equal probability. If we consider that all the extracted balls are arranged orderly in a line, then the winning ball B_n can be any of these balls, with equal probability. Thus, B_n is uniformly distributed on the set $\{1,2,\ldots,n\}$. This means that the probability that the highest number will be seen at extraction i is $\frac{1}{n}$. If we denote by $B_n= i$ the event that the winning ball is at extraction i (E_i), then we write mathematically as follows:

$$P(B_n=i)=\frac{1}{n}, \quad \text{for all } \ i\in\{1,2,\ldots,n\}. \tag{1}$$

The probability of success with the "k-strategy" depends on the moment when the winning ball is extracted. We denote by $P(W_{k,n}|B_n=i)$ the probability of winning with the "k-strategy" given that the winning ball was picked at extraction i.

Using the law of total probability, we can write

$$P(Win)=\sum_{i=1}^{n} P(\textit{winning ball is at } E_i)\times P(Win\,|\,\textit{winning ball is at } E_i).$$

In words, at each extraction of rank i, the probability of success depends on the event that the highest number is written inside the extracted ball.

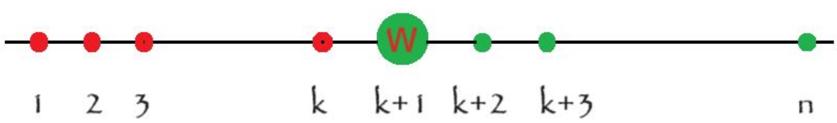
Mathematically, we write this as follows:

$$P(W_{k,n})=\sum_{i=1}^{n} P(B_n=i)\cdot P(W_{k,n}|B_n=i). \tag{2}$$

However, if the ball containing the highest number was already picked (but not accepted) at extraction $i \leq k$ (that is, this ball was among the k balls that we let go), then the "k-strategy" is a failure. Using conditional probabilities, we write this as

$$P(W_{k,n}|B_n=i)=0, \quad \text{for all } \ i\in\{1,2,\ldots,k\}. \tag{3}$$

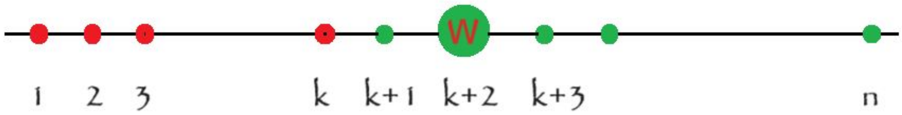
If the ball containing the highest number is the one extracted at rank $k+1$, then this ball has the highest number written inside among all the balls extracted so far. Based on the "k-strategy", the chief accepts this ball, and thus the "k-strategy" will be successful with probability 1. This choice is schematically represented in Figure 2. The red balls are the ones that the chief let go, the big green ball is the winning ball.



If the winning ball is the ball to be chosen from the urn at extraction $k+2$, then the "k-strategy" will be successful if and only if one of the first k balls (the ones that were automatically rejected) contains a number that is the highest among the first $k+1$ balls. With other words, among the first k 'let go' balls there is at least one number which is higher that the $k+1$ extracted ball. If that is the case, then the the $k+1$ extracted ball will not be accepted, and there are chances that the chief can win with the $k+2$ extracted ball.

There are k favourable cases and $k+1$ possible cases, thus the probability of winning at extraction $k+2$ is then

$$P(W_{k,n}|B_n=k+2)=\frac{k}{k+1}.$$



Using similar arguments, if the winning ball is the ball to be chosen from the urn at extraction $i>k$, with $i\in\{k+1,k+2,\dots,n\}$, then "k-strategy" will be successful if and only if one of the automatically rejected k balls contains the number that is the highest among the first extracted $i-1$ balls. In this way, there are k favourable cases out of $i-1$ possible extractions, and the probability of success given that the winning ball is drawn at extraction i is

$$P(W_{k,n}|B_n=i)=\frac{k}{i-1}, \quad \text{for any } i\in\{k+1,k+2,\dots,n\}. \tag{4}$$

Let us denote simply by $P_{k,n}$ the probability of winning using the "k-strategy" for an urn containing n balls. From the relations (1), (2), (3) and (4), we get that

$$P_{k,n}=\frac{1}{n}\cdot\sum_{i=k+1}^{n}\frac{k}{i-1}=\frac{k}{n}\cdot\sum_{i=k+1}^{n}\frac{1}{i-1}. \tag{5}$$

For $k=0$ we have that $P_{0,n}=\frac{1}{n}$, which is the probability of winning by choosing the first extracted ball.

Using the MATLAB software, we have calculated (see Table 1) and represented graphically (see Figure 1 and Figure 2) the exact probability $P_{k,n}$ for some given values of k or n.
In the particular case of $n = 73$, the graph of the exact probability for various values of k (given by formula (5)) is drawn in Figure 1. We observe (graphically) that this function is concave, with the maximum of this probability achieved for $k=27$ and the corresponding optimal probability is $P_{27,73}=0.3722$. For the neighbouring values of $k = 27$ we have obtained probability values lower than 0.3722. Indeed, $P_{25,73}=0.3715$, $P_{26,73}=0.3721$, $P_{28,73}=0.3718$, and $P_{29,73}=0.3709$.

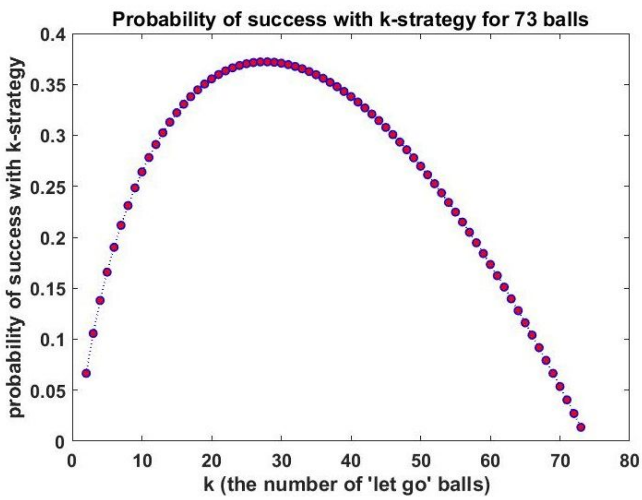


Figure 1. The winning probability as a function of k for $n=73$

Table 1 below contains the optimal value of k for the "k-strategy" and the corresponding probability $P_{k,n}$ for different values of n. We also observe that, for $n = 73$, the optimal value for k is 27, with the corresponding probability of winning $P_{27,73}=0.3722$.

Number of balls n	Optimal strategy k	Optimal probability
3	1	0.5000
4	1	0.4583
5	2	0.4333
6	2	0.4278
7	2	0.4143
8	3	0.4098
9	3	0.4060
10	3	0.3987
11	4	0.3984
12	4	0.3955
13	5	0.3923
14	5	0.3917
15	5	0.3894
16	6	0.3881
17	6	0.3873
18	7	0.3854
19	7	0.3850
20	7	0.3842
…	…	…
70	26	0.3724
71	26	0.3724
72	26	0.3723
73	**27**	**0.3722**

Table 1. The optimal value for k and the optimal probability of winning for various numbers of balls in the urn.

Figure 2 displays the values of k for the optimal "k-strategy" and the corresponding probability of winning $P_{k,n}$ as the number of balls in the urn is increased. We see that, as n increases, the values of k also increase and the optimal probability for the "k-strategy" decreases. We observe that the value of the optimal probability converges to a non-zero positive value.

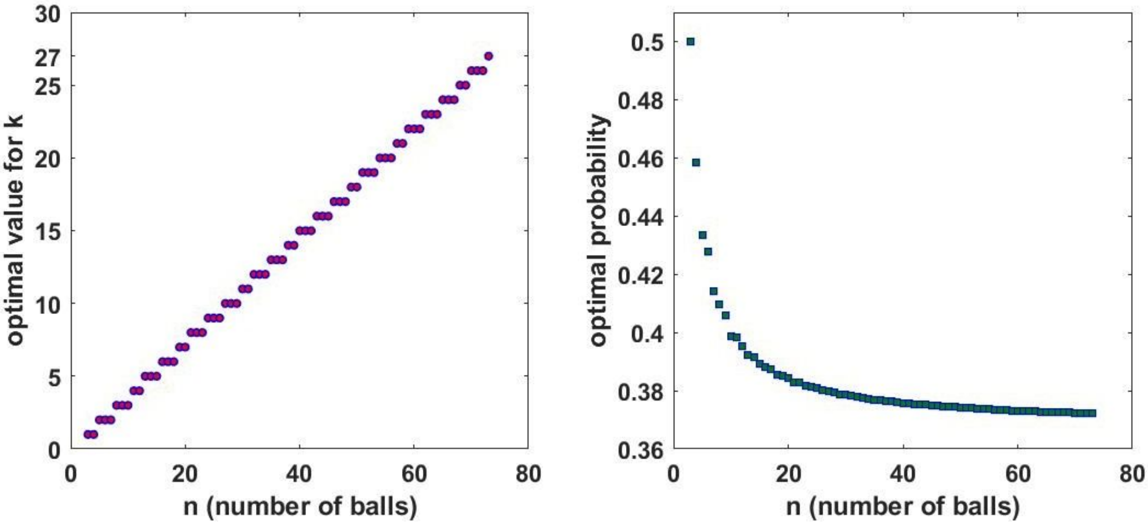


Figure 2. The evolution of the optimal value for k and the optimal probability of winning for various numbers of balls in the urn.

The analysis of the probability for success

Our next aim is to find a closed-form formula for the value of k (if that exists) for which the probability of success is maximum. As it is difficult to work with the sum in the formula of $P_{k,n}$, we try to find an approximate value of it. From the relation (5), for a fixed n, we can rewrite the probability of success as

$$P_{k,n}=\frac{k}{n}\cdot\sum_{i=k+1}^{n}\frac{1}{i-1}=\frac{k}{n}\cdot\sum_{j=k}^{n-1}\frac{1}{n}\cdot\frac{n}{j}. \tag{6}$$

Let us consider the following partition of the interval $\left[\frac{k}{n},1\right]$:

$$\frac{k}{n}<\frac{k+1}{n}<\frac{k+2}{n}<\ldots<\frac{n-1}{n}<1.$$

We observe that the following sum,

$$\sum_{j=k}^{n-1}\frac{1}{n}\cdot\frac{n}{j}=\frac{1}{n}\cdot\sum_{j=k}^{n-1}\frac{n}{j},$$

represents the left Riemann sum approximation corresponding to the above partition, of the integral

$$\int_{\frac{k}{n}}^{1}\frac{1}{x}dx=\ln(1)-\ln\left(\frac{k}{n}\right)=-\ln\left(\frac{k}{n}\right).$$

Therefore, by taking into account the expression (6), we get that, for n large enough, one can approximate the probability of winning using the "*k*-strategy" as follows:

$$P_{k,n}\simeq-\frac{k}{n}\cdot\ln\left(\frac{k}{n}\right).$$

Let us consider the function $f:(0,\infty)\rightarrow[0,1]$, given by $f(x)=-x\cdot\ln(x)$.

As we are trying to maximize the chances (the probability $P_{k,n}$), we want to find the maximum value for the function *f*. Firstly, we search for the critical points. As

$$f'(x)=-\ln(x)-x\cdot\frac{1}{x}=-\ln(x)-1=0,$$

we see that the unique critical point is $x=1/\mathrm{e}$.

As the second derivative of the function f at this point is equal to $-\mathrm{e}<0$, we conclude that $x=1/\mathrm{e}$ is a maximum point for *f*. Thus, the maximum value of the function f is $f(1/\mathrm{e})=1/\mathrm{e}\simeq 0.368$.

In words, the maximum probability of winning using the "*k*-strategy" for an urn containing n balls is obtained for the ratio $k/n\simeq 1/\mathrm{e}\simeq 0.368$, and the maximum value is also about 0.368, meaning about 37% chances of winning.

In conclusion, we should reject the first 37% of the balls in the urn, and then move on to the next best one. Thus, for if there are n balls inside the urn, the optimal number of balls that the chief will draw initially without stopping at any of them would be about n/e.

However, as n/e is not an integer, and also taking into account the fact that the formula (7) is just an approximation, one should check whether the number of "let go" balls from the urn is either $k=[n/\mathrm{e}]$ or $k=[n/\mathrm{e}]+1$ (Here, $[\cdot]$ represents the integer part of the number written inside the brackets.) This choice will depend on the value of *n* and, perhaps, it cannot be foreseen.

The particular case *n* = 73.

As we have seen earlier from the graphical representation, in the particular case of n = 73, the number of balls that we should reject following the "*k*-strategy" is k = 27. As

$$k=27=\left[\frac{73}{\mathrm{e}}\right]+1=\left[\frac{n}{\mathrm{e}}\right]+1,$$

we see that the above approximation for k is verified. Therefore, in this particular case n = 73, the approximation formula for k is $[n/\mathrm{e}]+1$. However, for other values of n (for example, n=3 or n=4), the better approximation for the number of "let go" balls turns out to be $[n/e]$.

After that rejecting the first 27 ball, regardless the value of numbers written inside them, we shall accept the ball with the highest number extracted till then. If this number exists at an extraction of rank higher than 27 and is the largest of all numbers, then our strategy is winning one. On the other side, if this number does not exist (that is, at any extraction of a rank higher than 27 we cannot find a number larger than the previous extracted numbers) (4), then the strategy is a failure.

3. Conclusion

In this paper, we propose to the chief the following "k-strategy" to achieve his goal. The optimal strategy for an urn having 73 balls is as follows: the chief must reject out of hand the first 27 balls, and then select the first ball (if it appears) that has a number inside that is higher than all of the numbers that were rejected before.

For a general n (number of balls in the urn), the strategy is as follows. The chief must let go the first $k \simeq n/e$ extracted balls, without accepting any of them (regardless the value of numbers written inside them), and then, for the subsequent extractions, he must accept the ball that contains the highest number among all the numbers extracted so far, if it exists.

If such a number appears during any extraction of a rank higher than k and is equal to the maximum among all n numbers, then the chief wins. Otherwise, the chief loses.

For large n, we have found that the optimal value for k in the "k-strategy" is is an integer close to n/e. It can be either $k=[n/e]$ or $k=[n/e]+1$, depending on the particular choice of n (both values have to be checked). This means the chief has to wait until about 37% of the balls have been extracted, and then he must select the first ball with the highest number among the extracted ones, if it exists. The chances of success are also about 37%.

References

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2. Tom Ferguson, *Who Solved the Secretary Problem?,* Statist. Sci. 4(3): 282 289 (1989).
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4. The MathWorks Inc., MATLAB R2022a, Natick, Massachusetts: The MathWorks Inc. https://www.mathworks.com (2022).
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Editing notes

(1) Readers familiar with basic probability concepts can skip this section.

(2) This is for the case of a finite probability space. In general, an event with probability 1 is said to occur *almost surely,* and an event with probability 0, *almost never*.

(3) We assume that the extraction continues … even when *some ball* is accepted, wether it is winning or not. This does not change the probabilities of extractions up to this ball.

(4) Or if that number exists, but there's a larger number among those who haven't yet been extracted.