Solving Traffic Jams

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1 Problem Statement

In order to simplify the problem of traffic jams, let's start with a simple case:

- a single line of cars
- the cars are all identical and move at the same speed
- two possible positions: stop or go
- a car moves forward one square when the space in the front of it is empty
- a car stays in place when the space in front of it is occupied.
- by placing a number of cars, study the evolution of traffic

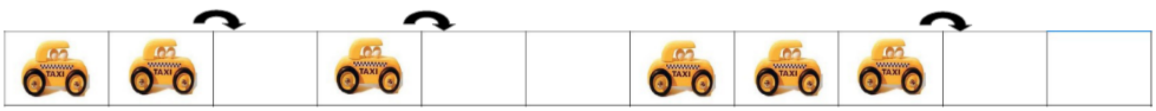

Figure 1: Easy problem visualisation

Our goal is to study the traffic flow and find out a general formula that expresses in how many steps the traffic will become fluid. We consider a traffic fluid when every single car can move forward.

The traffic jams are a real problem in today's world. Not only because of the time lost by every single person in a traffic jam, but because every car is polluting the atmosphere. Knowing this facts, other similar articles were published (https://www.mdpi.com/2076-3417/9/14/2918) in order to solve this problem. We approached this problem differently and we tried to use an algorithmic approach. This approach will produce a result based on the number of cars in each case. We started from more particular cases and, finally, by using mathematical induction, we believe we reached some formulas.

2 Explaining the Notations

In order to simplify the process of solving the problem of Traffic Jams we agreed that it would be better to create some notations:

- 1 stands for one car in the traffic
- 0 stand for an empty space in the traffic

After implementing this notations we can recognise better the patterns in the Traffic Jams and start solving our problem.

3 Cases of the Problem

By advancing in the research of our problem, we discovered that there are 2 main cases that need to be solved:

- the first case is when we have a row of cars and there is no space between them

1111111...1
n cars

- the second case is when we have a row of cars and there are spaces between them

111...1000...0111...1000...0111...1...
n cars, m spaces

Our goal is to discuss both cases, starting from more simple cases.

3.1 The First Case

We can assume that the simplest case is when we don't have any spaces between the cars. In this case our Traffic Jam will look like this:

1111111...1
n cars

Let's take, in the beginning, an even simpler case, with just 5 cars:

11111
111101 – after 1 step
1110101 – after 2 steps
11010101 – after 3 steps
101010101 – after 4 steps
Result: in 4 steps the traffic will become fluid

Another example for the first case would be the one with 7 cars:

1111111
11111101 – after 1 step
111110101 – after 2 steps
1111010101 – after 3 steps
11101010101 – after 4 steps
110101010101 – after 5 steps
1010101010101 – after 6 steps
Result: in 6 steps the traffic will become fluid

As we can observe, in the first case we had **5 cars** and the traffic will become fluid in **5-1 = 4 steps**. In the second case with **7 cars** we observe that the traffic will become fluid in **7 - 1 = 6 steps**.

In order to generalize we will take now an example with n cars in the row.

1111111...1
111111...101 – after 1 step
11111...10101 – after 2 steps
....
101010101...01 – after n - 1 steps
Result: in n-1 steps the traffic will become fluid

In conclusion, for the simple case we believe that we find a formula that can show us the number of steps that are necessary to be done before a traffic jam to be fluid.

3.2 The Second Case

We have to understand that in real life we don't always have a traffic jam that looks like

1111111...1.
n cars

In order to understand how the real traffic works, we developed and studied the second case, where we have two groups of cars separated by some empty spaces:

111...1000...0111...1
X_1 Y_1 X_2

In order to have a better understanding of this case, we decided to create new notations:

- X_1 = the number of cars from the first group
- X_2 = the number of cars from the second group
- Y_1 = the number of empty spaces between the two groups

Now we can move forward and analyse some particular cases. In the first case we will study a traffic jam formed by seven cars:

111001111
1101011101 – after 1 step
10101110101 – after 2 steps
010111010101 – after 3 steps
0011101010101 – after 4 steps
00110101010101 – after 5 steps
001010101010101 – after 6 steps
Result: in 6 steps the traffic will become fluid

Let's take now another example with seven cars also, but with more empty spaces between the two groups.

11100001111
110100011101 – after 1 step
1010100110101 – after 2 steps
01010101010101 – after 3 steps
Result: in 3 steps the traffic will become fluid

In order to generalize we will take now an example with two groups of cars divided by a group of empty spaces. The notations will be X_1, Y_1 and X_2.

111...1000...0111...1
X_1 Y_1 X_2

In order to reach the generalization of our problem, we have to see how the jam looks like after Y_1 steps.

111...1 0101...01 111...1 – after Y_1 steps
$X_1 - Y_1$ Y_1 $X_2 - Y_1$

Now let's analyse the result:

- in the first group of cars there are only X_1-Y_1 cars left
- after the first group of cars comes Y_1 pairs of 01
- in the second group of cars there are only X_2-Y_1 cars left
- after the second group of cars comes Y_1 pairs of 01 which can be ignored, because they will not influence anymore the traffic

Now that we reached this point we believe that there are two cases of our problem:

1. if $Y_1 >= X_2$, then the jam can be shortened, because everything after the 1st group will be just pairs 01 that don't influence anymore the traffic. In this case the row will look like 11111…1 ($X_1 - Y_1$ times) and from the simple case we know that this is resolved in X_1-Y_1 - 1 steps. To sum up, the result will be: $X_1 - Y_1 - 1 + Y_1 = X_1 - 1$ steps.
2. If $Y_1 < X_2$, then there are still some cars left in the second group and this is certainly going to influence our final answer. The jam can be shortened because everything after the second group will be just pairs 01 that don't influence anymore the traffic. In this case, our row will look like 111…1 0101…01 111…1. Firstly, we believe that we have to calculate in how many steps does the second group can be solved and we can find the answer from the simple case, which is $X_2 - Y_1$ steps. At this number we have to add the number of cars that are behind the second group. To sum up, the result will be: $X_2 - Y_1 - 1 + X_1 - Y_1 + Y_1 + Y_1 = X_1 + X_2 - 1$.

These examples made us believe that the number of empty spaces between the two groups of cars influence the final result. Studying numerous cases, we believe that:

- if $Y_1 >= X_2$, then the result will be $\max(X_1, X_2) - 1$
- if $Y_1 < X_2$, then the result will be $X_1 + X_2 - 1$

4 Conclusions and Future Plans

During this study year, we were able to discover some formulas which help us to understand better how the traffic jams work. Still, there is much more in this area of expertise for us to explore and we will continue to search for a more general formula, which would indicate the number of steps until the traffic becomes fluid for any type of traffic configuration.