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Guidelines for assessing students' skills developed through math research

Handbook for teachers

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Introduction

The present guidebook has been produced within *Learning math and languages through research and cooperation – MatLan*, a 2-year Erasmus+ Strategic Partnership project (2014-1-RO01-KA201-002699) implemented with the support of the European Union by two secondary schools: one in Romania and one in France. Within the MatLan project, the two partner schools worked together to organise and carry out research workshops for upper secondary school students.

The math research workshop for students replicates research activities carried out by professional researchers. The workshop capitalises on the students' inventiveness and creativity, inviting them to discover mathematics (and computer sciences) and to carry out research work in this field. Throughout an academic year, working in small groups, the students look for mathematical solutions and do computer modelling for a problem/ research topic launched by researchers in the field of 'hard' sciences. The students' activity is facilitated by a teacher and supported by a professional researcher. After the research activities proper are over, the students seek opportunities to share the results they have found – for instance, in scientific conferences, students' congresses, etc. In addition, the students write research report articles in which they share the research topic, how they approached the solution to the problem and the results they found. The research report articles authored by students are published in the website mathenjeans, after they are reviewed by an editorial board.

One of the significant outputs of the project is the present guidebook, developed first of all for teacher who run math research workshops. The guidebook complements another intellectual output of the project, "Syllabus for the elective course *Mathematics research workshop/ Studying math through research*", in that it reveals how the project team operationalised two transversal competences and a specifically mathematical competence, preparing for the development of the methodology (methods and tools) for assessing them. The guidebook addresses teachers and experts in didactics who are interested in developing competence assessment tools. We are confident that the process of developing the competence assessment methodology / instruments described in this guidebook may prove useful for specialists interested in competence assessment.

The guidebook is structured in four chapters. The first chapter presents aspects connected to assessment in the mathematics research workshops – what we know about how assessment is done in such research workshops, why we aim to assess competences students develop in the research workshops, and some aspects that should be kept in mind when assessing competences. In the second chapter, we share the diagram of competences students develop in the research workshops, and operationalise / define the three competences students develop in these workshops: *collaborative problem solving*, *use of aids and tools,* and *written and oral communication skills for sharing the research results.* Chapter three includes the methodology of assessing the above-mentioned competences, which was tested over the period of an academic year, and then revised. The last chapter shares the conclusions we drew upon testing the assessment methods and tools, as well as a few ideas related to how our approach can be continued.

We would like to express our thanks to all those who have supported us in preparing this guidebook:

- Mathematics teachers of Colegiul Național Emil Racoviță Cluj-Napoca and of Lycée d'Altitude de Briançon, who tested the assessment methods and tools developed in the 2015-2016 academic year;
- French language teachers of Colegiul Național Emil Racoviță and English language teachers of Lycée d'Altitude de Briançon, who supported us in preparing the tri-lingual glossary (English – French – Romanian);
- Experts of the MATh.en.JEANS Association, the ANIMATH Association, the Romanian Reading and Writing for Critical Thinking Association, and participants in the events "Through research, mathematics becomes attractive" (17 March 2016 – Cluj-Napoca), "Mathematics Education in a Connected World" (16-21 September 2015 – Catania), "The 8th Science Projects Workshop in the Future Classroom Lab" (17-18 October 2015, Brussels), Scientix National Conference (30-31 October 2015 – Bucharest), who provided us valuable feedback and thus contributed to the development of the present publication.

We would be happy to know that this guidebook has proven useful to you. If you wish to share your feedback with us, please contact us; our contact details are available on the project website.

The Authors

1. Assessment in the math research workshops

In the introduction we described briefly what a mathematics research workshop is – an ampler description is available in the Syllabus for the elective course *Doing Maths as Researchers Do It* (Văcăreţu & Proal, 2016, pp. 10-14).

Math research workshops have been carried out in France since 1989, under the name MATh.en.JEANS (MeJ). As a rule, at the end of each academic year, each MeJ workshop coordinating teacher prepares a report on the activities carried out within the workshops. As far as we know, the teachers do not assess the students' learning within the workshops.

Annually, the MATh.en.JEANS Association carries out an evaluation of the activities within the MeJ workshops, using questionnaires (for students, for teachers and for researchers); the information gathered by means of questionnaires together with the numerical data (number of students, number of teachers, number of research workshops, number of research topics, etc.) collected via the website MATh.en.JEANS are presented in the annual impact reports (the report for the 2014 – 2015 academic year is available at http://www.mathenjeans.fr/sites/default/files/bilan_impact_mej_2015.pdf).

The information gathered with the help of student questionnaires relates to: reasons for enrolling in the research workshop, participation in the MeJ annual Congress, the students' opinions about: knowledge of the way in which scientific research is done, knowledge of the profession of scientific researcher, knowledge of the domains of math application, enhanced desire to study sciences, enhanced desire to learn mathematics, changes in the level of concentration in class, changes in the students' perceptions of the teacher who facilitates the math research workshops (MATh.en.JEANS, Questionnaire élèves - MeJ 2015/2016, 2016). The questionnaire for teachers includes several sections: personal information, perceptions about the MeJ experience in the current academic year, opinions about the students' participation and progress, opinions about the MeJ Congress, opinions about knowledge of a researcher's profession and guiding students to study sciences in higher education, opinions about management of MATh.en.JEANS (MATh.en.JEANS, Questionnaire Enseignant.e.s - MeJ 2015/2016, 2016). As mentioned, the students' progress is assessed by means of the questionnaire for teachers; below, we share a few questions that relate to this aspect:

- Have your students improved their understanding of mathematical tools? (choices for answering: yes/ no/ I don't know)
- In your opinion, have your students improved their oral and written expression? How about their capacity to synthesise information, ideas, etc.? (choices for answering: yes/ no; comments)
- In your opinion, do the students feel more comfortable speaking in public? (choices for answering: yes/ no)
- Are you pleased with your students' capacity to present the results of their own research in the MeJ Congress? (choices for answering: yes/ no/ it depends)
- Are you pleased with your students' capacity to answer questions posed by the audience in the Congress? (choices for answering: yes/ no/ it depends)

- In your opinion, what was the general impression about your students (*our note*: this refers to the students' participation in the MeJ Congress)? Strengths, weaknesses? (open-ended question). (MATh.en.JEANS, Questionnaire Enseignant.e.s - MeJ 2015/2016, 2016)

Obviously, this assessment provides information about the impact of MeJ workshops, in general. This is necessary. Is it, however, also sufficient?

1.1. Why should we assess the competences students develop in the mathematics research workshops?

In France, the mathematics research workshops are carried out as extra-curricular activities or they are included in the personalised support lessons for students. In both situations, the teachers **don't** grade the students who participate in MeJ. In Romania, up to June 2016, only two math research workshop had been organised, as extra-curricular activities. We know that assessment and grading are two different things – but they are connected. If we need not grade students, why should we assess their competences developed within the MeJ workshops?

Mathematics research workshops are learning activities, where students actually learn – in a different manner than they do in lessons, but they do learn. They develop certain competences. Șerban Iosifescu was writing in an article, "We cannot improve that which we cannot assess" (Iosifescu, 2015). How can we establish the effectiveness of the research workshops if we do not assess what the students learn? How can we decide what the students learn if we do not assess the competences they develop in the mathematics research workshops, especially since math research workshops are **not** mainly about new knowledge? Learning in the math research workshops focuses on the development of skills and attitudes needed for learning math and sciences/ computers sciences, but also for other contexts (for instance, mastering specific work techniques needed in higher education). For this reason, we believe that learning in the math research workshops aims to develop competences and, as a natural consequence, assessment should target the competences developed. Assessment in this context does not aim to rank students by establishing what level of mastery of a particular competence they have reached. Assessment aims to establish whether the research activity that we invite the students to carry out contributes to competence development, which competences have been developed and what is the size of the students' progress.

Competence assessment is assessment for learning. Assessment for learning is the process of seeking and interpreting evidence, which supports the student and the teacher in deciding where the student is in the process of learning, where s/he should get and how best to get there (Assessment Reform Group, 2002). In order for assessment to support learning, we must articulate for the students the nature of the expected outcomes of learning, the competences we expect them to master, and what it means to have developed a particular competence (providing examples if necessary).

1.2. How we view assessment in the mathematical research workshops

We will list below the major aspects we think we should keep in mind when we aim to assess competences students develop in the math research workshops:

- Assessment is aimed at learning, allowing students to observe the evolution of their own competences, in other words, this type of assessment is of **formative** nature, it is **assessment for learning**.
- Both students and teachers should be engaged in the assessment process. We encourage the use of self-assessment and peer assessment, which support the students' understanding of how they learn and of the process they undergo in the development of their competences. In this way, we believe that assessment also contributes to **learning how to learn**.
- For carrying out the assessment, the competences need to be operationalised and the students need to understand the behaviours that demonstrate that a certain competence has been mastered. In this manner, **assessment provides a clear description of the expected results**.
- We should not fall into the trap of thinking that anyone can assess themselves or someone else (e.g. a peer). David Pepper has conducted some studies that reveal that students need to learn how to self-assess or assess peers (Pepper, 2013, p. 22). We are aware that although students are honest and well-meaning when they self-assess or assess their peers (for the purpose of supporting learning), they need to be **prepared for this evaluative approach**. They need to understand the expected results of learning, the assessment tools that they are going to employ and to practice – under guidance – the use of the various assessment methods.
- We are mainly interested in the approach and the process that students go though as they carry out the research, and not necessarily the solutions to the problem. Consequently, this is **process-assessment.**
- Assessment should **generate information that is easily understandable for students and teacher,** and provide useful feedback for the improvement of learning and for new learning opportunities.
- **Assessment should be valid**, in other words, there needs to be coherence between how assessment is planned and how the results of the assessment are used (Pepper, 2013).

1.3. Our approach to the development of the assessment methodology

In order to develop the methodology for the assessment of competences students develop in the research workshops, we moved through the following stages:

- Identification of the competences students develop in the research workshops;
- Selection of three competences the development of which we plan to assess in the following 3-4 years;

- Operationalising the competences (or making them visible), i.e. identification of the sub-skills that build up the aforesaid competence, and the behaviour that demonstrates that a person had developed those sub-skills;
- Description of three levels of performance (novice, competent, expert) for each behaviour indicator of each competence;
- Elaboration of the developmental progression (for each competence;) the developmental progression is conceived as a detailed "roadmap" which illustrates increasingly sophisticated behaviour which individuals display as they progress from the level of novice to that of an expert in any domain of learning (University of Melbourne, 2015);
- Selection of the assessment methods and tools using the developmental progression (for each competence).

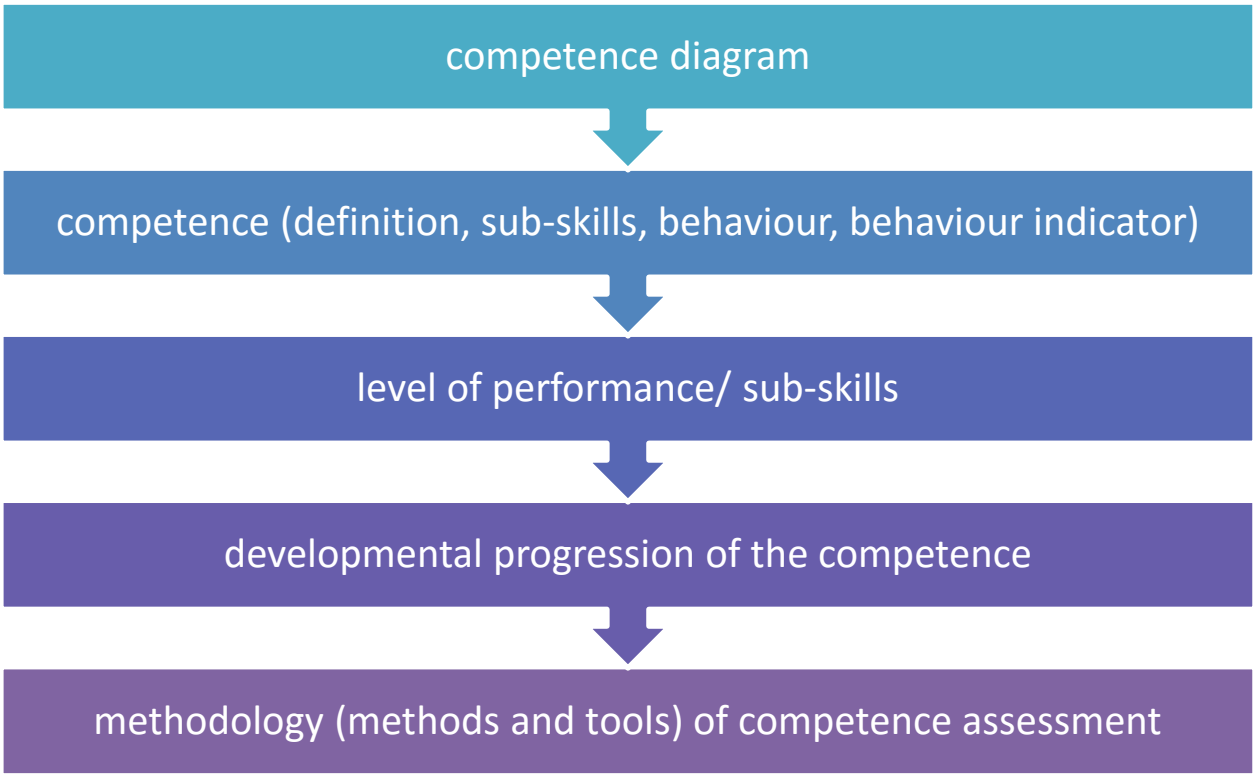


Figure 1. The process of developing the methodology for assessing the competences students develop in math research workshops

2. Competences developed by students in the mathematical research workshops

Through literature review we identified several definitions of competence. In our approach, we have decided to rely on the definition formulated by Hoskins and Deakin Crick (Hoskins & Deakin Crick, 2010) included in the *KeyCoNet 2013 Literature Review* (Arjomand, et al., 2013): by competence we mean the combination of knowledge, skills, understanding, values, attitudes and desire (to do), which lead to an individual's effective action in a particular domain. In this definition, the focus is on two elements: **carrying out a complex real-life task,** on the one hand, and **the multiple dimensions of knowing** – for instance, know how to do something, knowing yourself, knowing your desires, knowing why something is important, knowing about something, etc. These multiple dimensions of knowing are similar to the four pillars of knowledge: learning to know, learning to do, learning to live together with others, and learning to be (Delors, 1996). Competences only show in action, which is why action (or the manner of doing something) is very important. Moreover, since a competence is identifiable in an authentic life context, developing competences relies on real-life experiences and takes into account the whole spectrum of learning opportunities (informal, non-formal, formal).

Students enrol voluntarily in the math research workshops. Naturally, in this context, they are not selected based on a test or based on their mastery of mathematics. Their systematic activity over the weekly workshops, but also beyond them, capitalised on through their participation in scientific conferences, allows modestly performing math students to take a different perspective on math, and provides high-performing students the opportunity to deepen their knowledge. Both individual and team research – sometimes of a clearly experimental nature for the sake of identifying solutions to the research problem – brings significant changes to the relationships among students and between students and teachers. In the absence of time constraints – as in this context the teacher doesn't teach any specific content – work methods and tools become a necessity to advance in studying the topic and favour the development of specific scientific research competences.

After observing the students' activity in the research workshops, analysing the tasks the students had had the workshops, and studying the relevant literature, we concluded that the frame of researcher evolution (Vitae®, 2011) is the most appropriate starting point for defining the competences students could develop in the math research workshops. We decided on four domains:

- *Knowledge and intellectual abilities,* a domain which includes 3 subdomains:
    - Mathematical competences – i.e. competences specifically pertaining to mathematics as a discipline;
    - Creativity – i.e. the ability to produce a new or original solution to an unusual, non-trivial problem (Sriraman, 2009);
    - Cognitive skills – i.e. core skills your brain uses to think, read, learn, remember, reason, and pay attention; each of them plays an important part in processing new information (LearningRx, Inc, 2003-2016).

- *Collaboration* – this domain includes the competences/ skills needed by people who work together, share ideas and results to solve the same (research) problem.
- *Communication* – this domain includes skills/ competences needed for sharing information orally, in writing, etc.; in the research workshops, this means, on the one hand, communication within the group/ groups of students who research the same topic (internal communication), and, on the other hand, communication of the research results to third parties (external communication). In both situations, communication means shifting from everyday language to formal symbolic language (and *vice versa*), sharing the conclusions of evaluations of both processes and outputs, clear and accurate expression in oral and in written form.
- *Personal qualities* – this domain includes: enthusiasm, perseverance, integrity, self-confidence, introspection, responsibility.

In the diagram below we have included all the domains and competences / skills that students develop in the research workshops. A special situation is that of *collaborative problem solving* and *communication in, with and about mathematics*, competences that are positioned at the borderline between two domains.

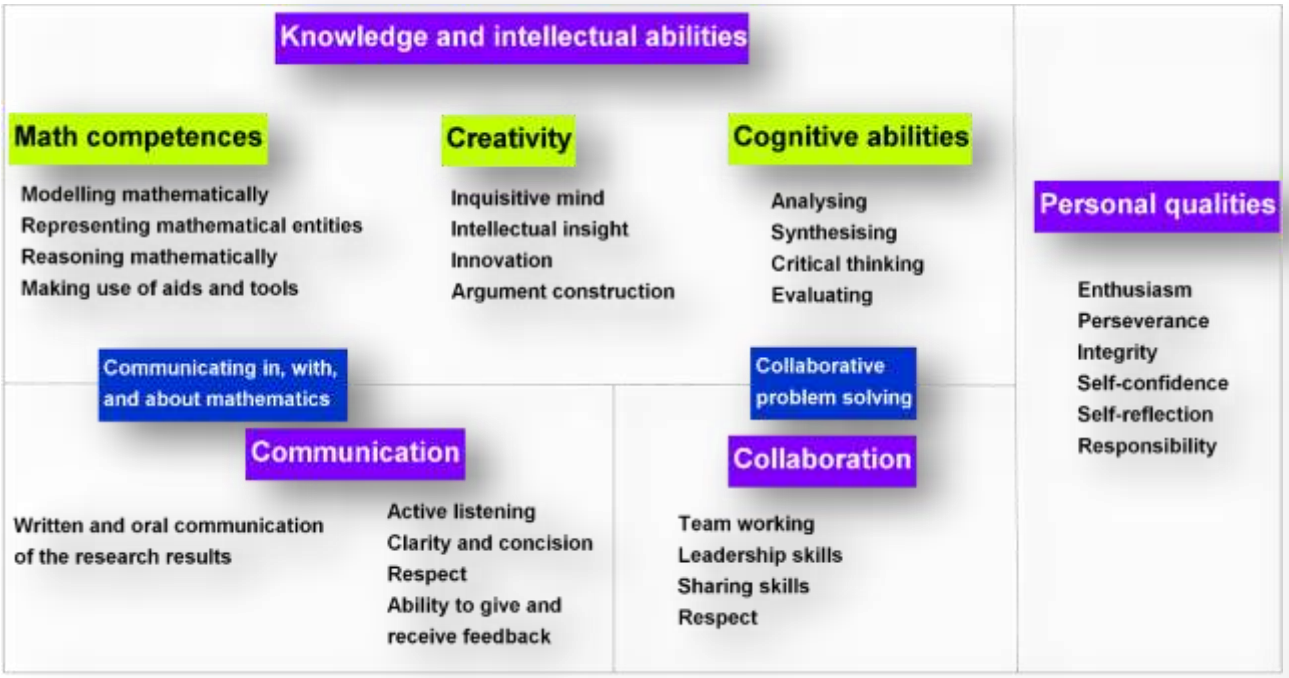


Figure 2. Diagram of competences students develop in the mathematical research workshops

In the following, we will define and operationalise three of the competences students develop in the MeJ workshops: two transversal competences: *collaborative problem solving* and *written and oral communication of the research results,* and a mathematical competence: *use of aids and tools*.

2.1 Collaborative problem solving competence

2.1.1. Definition of the competence

Collaborative problem solving involves two or more people working towards a joint problem solution. All the participants must contribute their resources, skills, etc. so the problem can be solved. Those two or more people will have a common goal; the resources needed to solve the problem are beyond the capacity of either person alone. If people work together, they might be able to work it out. Collaborative problem solving, or working with others to solve a common challenge, includes the contribution and exchange of ideas, knowledge, or resources in order to achieve a shared goal.

This competence brings together two skills: problem solving - which can be included in the category *Knowledge and intellectual abilities*, related to ways of thinking (Blinkley, et al., 2012); and collaboration – which, in our model, is a category by itself, related to ways of working (Blinkley, et al., 2012).

In the MATh.en.JEANS workshop, mathematics research topics are launched by professional researchers. Small, 2-3 student groups, choose one of the proposed problems and do research work to solve it. The students have to organize their work, identify the resources (strategies, knowledge, experience, equipment, software, materials); decide how the resources will be used for building and maintaining a shared understanding of the task and its solutions.

While implementing the MATh.en.JEANS workshop, we identified the following steps the group of students take for solving the problem:

Understand the problem	• Each student explores the problem space[1] and identifies the elements and aspects of it. • In the group, students collect and share information about the problem elements and how they are linked. In this stage, the group of students defines the problem space collaboratively.
Devise a plan	• In the group, students discuss; they look for patterns and links between elements of the problem, analyse particular cases, organize and categorize the information/ data, both within and across the individual areas of expertise; students re-formulate the problem in a familiar/ mathematical language and plan their approach for solving the problem.
Carry out the plan	• In the group, through discussions and collaboration, the students set up procedures and strategies to solve the problem. They begin to formulate rules or contingencies associated with actions and observations (“if …, then”). These rules or contingencies lead to generalisations.
Look back and check	• Students test hypotheses by challenging generalisations, check process and solutions.

Table 1. MATH.en.JEANS approach to collaborative problem solving

[1] The problem space is the way an individual or a group of individuals think about the transformation between the beginning stages of the problem and throughout finding a solution.

The *problem-solving component* of the Collaborative problem-solving competence

The major stages of the problem-solving process identified by Polya (1973) are appropriate for our research approach: understanding the problem, devising a plan, carrying out the plan, looking back (examining the solution obtained and checking the results) – see Table 1. Both inductive and deductive reasoning are embedded within this process, as the research task challenges the students to detect information, identify patterns or analyse particular situations (as part of the inductive process), and then identify rules and test hypotheses (as part of the deductive process).

The *collaboration component* of the Collaborative problem-solving competence

Patrick Griffin (2014) describes two ways of looking at the collaboration component within the *collaborative problem-solving competence*:

a. as a target skill – in this case collaborative problem solving is curriculum independent, based on the development of skills other than cognitive (math competences – within the MATh.en.JEANS context) embedded within the school curriculum defined for each subject, but still domain specific. In this case, collaboration is important because collaborative problem solving requires team work, reaching agreements on ideas or hypotheses to be tested and on the way the team will proceed;

b. as an enabling skill – in this case, the collaborative problem-solving competence is a facilitating skill to enable cognitive-based skills to be acquired. Collaborative problem solving is curriculum embedded and subject related, so collaboration will be secondary to the students' understanding and skill development related to the subject.

Within the context of the MATh.en.JEANS workshops, we view the collaborative problem solving skill as a target skill because:

- the math research topics are not related to a specific math curriculum (neither country specific, nor age-group specific); however, the collaborative problem solving (research) task is specific to the math-science-technology domain;
- within the research work, collaboration is essential – e.g. in our experience, when first confronted with the research topic, individuals had no idea from where to start. After discussing with their peers, ideas about how to start their research emerged. We actually engage the students in their zone of proximal development. The zone of proximal development is the difference between a student's actual developmental stage and his or her potential developmental stage when collaborating with a more able peer or teacher (Vygotsky, 1978) – see figure 3. Students on the MATh.en.JEANS teams have different skills, different knowledge and each of the students can be the 'knowledgeable other' in different moments of their work.

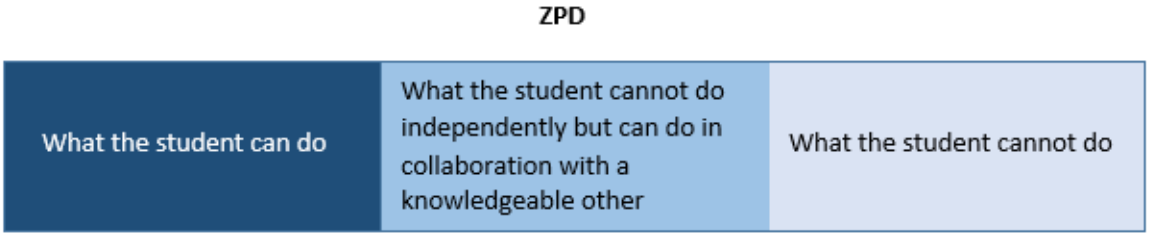


Figure 3. Diagram of the Zone of Proximal Development (ZPD)

Remark

For the first time, OECD will include in its 2015 PISA test the assessment of collaborative problem solving. The definition of Collaborative Problem Solving for PISA 2015 is defined as follows (PISA 2015, 2013):

Collaborative problem solving competency is the capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills and efforts to reach that solution.

The elements of the above definition are similar to the ones we considered in our definition:

- collaborative problem solving involves an individual's cognitive processing that engages both cognitive and social skills;
- the focus is on the collaborative actions the students engage in while trying to solve the problem.

2.1.2. Make the competence visible

After observing what students do, make, say and write while researching the math topics in the MATh.en.JEANS workshop and analysing the definition of collaborative problem solving skills (ATC21S Project, 2012) and Griffin's framework of both, cognitive and social skills (Griffin, 2014), we adapted Griffin's frameworks to the MATh.en.JEANS context.

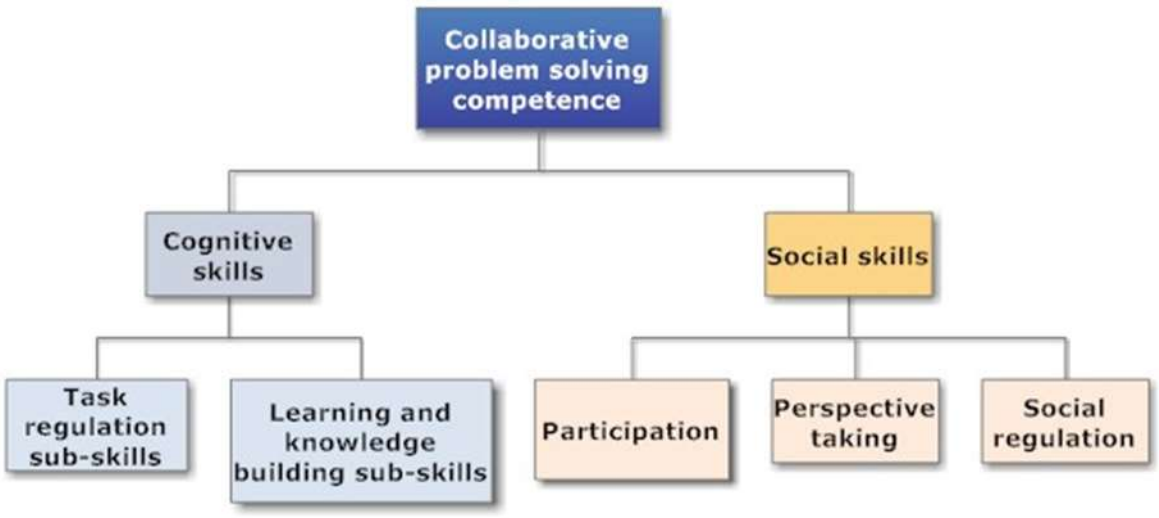


Figure 4. Diagram of the skills and sub-skills in the Collaborative problem solving competence

Cognitive sub-skills of the Collaborative problem solving competence

The cognitive sub-skills in collaborative problem solving are similar to those needed in individual problem solving and they refer to the ways in which students manage a task in progress (task regulation skills) and the reasoning skills they use (the learning and knowledge-building skills).

Cognitive sub-skills	Behaviour	Behavioural indicator (Griffin, 2014)
Task regulation skills		
Problem analysis	Student identifies the elements of the task/ problem space and information available for each element; they look for patterns and links between elements of the problem, analyse particular cases.	Analyses and describes the problem in familiar mathematical language.
Goal setting	Student formulates and shares specific sub-goals that will help to monitor the process of collaboration progress towards problem resolution. The goal may take the form of rules (if I do A, then B should happen and I'll be able to prove C/ make progress towards the goal C).	Sets a clear goal for the task.
Learning and knowledge building skills		
Relationships	Student recognises, represents and formulates relationships and patterns in the data – this is important when the students attempt to build an information base to define the problem space collaboratively and to identify links between actions and consequences, between observations and patterns.	Identifies connections and patterns between and among elements of knowledge.
Cause and effect ("if … then …")	Student identifies and communicates cause and effect and this allows the group to establish simple rules which students share in order to make progress towards a solution. When students observe cause and effect systematically, they discuss the potential of rules and establish rules either for the regulation of the task or in manner of collaboration. Student observes and records information and consequences.	Uses understanding of cause and effect to develop a plan/ strategy.
Hypotheses ("what if …")	Student demonstrates an ability to generalize by forming and testing hypotheses, using a what-if approach. Students propose one another suggestions as: "what about we try this …."/ "what do you think could have happened if we had used another approach".	Adapts reasoning as information or circumstances change.

Table 2. Framework of cognitive skills for collaborative problem solving in MATh.en.JEANS

Social sub-skills of the *Collaborative problem solving competence*

Students' social skills are very important in MATh.en.JEANS workshops for collaboratively researching the math topics. Within the social skills domain, Griffin (Griffin, 2014) identified three sub-skills: participation, perspective-taking skills (the ability to see a situation from the viewpoint of another person) and social regulation skills (the ability to deal with different viewpoints and different opinions).

Social sub-skills	Behaviour/ what does it mean	Behavioural indicator (Griffin, 2014)
Participation skills		
Action	It is defined as the general level of a student's participation, whether coordinated with other students' efforts or not.	Activity within environment.
Interaction	It refers to the capacity to respond to, or coordinate with others, ranging from answering a question to actively initiating and coordinating efforts or prompting others to respond.	Interacting with, prompting and responding to contributions of others.
Perspective taking		
Adaptive responsiveness	Student integrates contributions of peers into his/ her own thoughts and actions.	Ignoring, accepting or adapting contributions of others.
Social regulation		
Negotiation	Student finds ways to reconcile different perspectives and opinions, and to accommodate differences.	Achieving a resolution or reaching compromise.
Self-evaluation	Student evaluates his/ her own knowledge, strengths or weaknesses in the problem solving context.	Recognising own strengths and weaknesses.
Responsibility initiative	Students work in different ways to take joint accountability and engagement within a collaborative context. Some focus mainly on their individual tasks, while others work on a shared problem representation, a strategic plan towards a solution, and engage in regular monitoring of the group's progress.	Assuming responsibility for ensuring parts of the task are completed by the group.

Table 3. Framework of social skills for collaborative problem solving within MATh.en.JEANS

2.1.3. Definition of the criteria for each behavioural indicator

In order to define the quality criteria for each behavioural indicator, we chose to describe three levels of performance for each of the behavioural indicators. In describing these levels, we took into consideration the observations made during the working process within the MATh.en.JEANS workshop and the reviewed literature: (ATC21S Project, 2014) and (Griffin, 2014).

Subskills	Behavioural indicator	Low (novice)	Medium (competent)	High (expert)
Task regulation skills	Analyses and describes the problem in familiar language.	Describes the problem by reading it aloud. Explores 1 – 2 particular cases.	Describes the problem by stating the given situation in his/ her own words. Divides the problem into subtasks. Explores 3 – 4 particular cases by using non-mathematical models (e.g. manipulatives).	Describes the problem space in a familiar mathematical language and states the hypotheses and the conclusion of the problem. Set/ define the logical sequence of subtasks. Explores more than 4 particular cases by using ICT (e.g. Geogebra) and/ or manipulatives.
	Sets a clear goal for the task.	Sets general goal, such as task completion.	Sets goals for subtasks.	Sets goals that recognise relationships between subtasks.
Learning and knowledge building skills	Identifies connections and patterns between and among elements of knowledge.	Focuses on isolated pieces of information.	Links pieces of information.	Identifies patterns in approaching the research task.
	Uses understanding of cause and effect to develop a plan/ strategy.	Activity is undertaken with little or no understanding of consequence of action.	Identifies sequences of cause and effect.	Plans strategy based on a generalised understanding of cause and effect.
	Adapts reasoning as information or circumstances change.	Tests hypotheses.	Modifies hypotheses.	Reconstructs and reorganizes understanding of the problem.

Subskills	Behavioural indicator	Low (novice)	Medium (competent)	High (expert)
Participation skills	Activity within environment.	No or very little participation.	Active if supported.	Active without being supported.
	Interacting with, prompting and responding to contributions of others.	Acknowledges communication but does not provide information or resources.	Responds to prompts in interaction (e.g. provides information and resources to peers).	Prompts interaction (e.g. initiates and promotes interaction with peers).
Perspective taking skills	Ignoring, accepting or adapting contributions of others.	Ignores contribution of peers.	Responds to contributions of peers (e.g. considers contribution but doesn't make changes).	Incorporates contributions of peers to suggest possible solution paths (e.g. makes changes based on contribution of peers).
Social regulation skills	Achieving a resolution or reaching compromise.	Reaches a common understanding.	Comments on differences but does not achieve resolution.	Achieves resolution of differences.
	Recognising own strengths and weaknesses.	Is aware of own performance.	Comments on own performance.	Evaluates own performance.
	Assuming responsibility for ensuring parts of the task are completed by the group.	Undertakes activities largely independent of others.	Reports to others on progress of activities.	Assumes group responsibility as indicated by use of 'we' rather than 'I' or 'you'.

Table 4. A framework of low, medium and high level cognitive and social skills for collaborative problem solving

2.1.4. Developmental progression

The developmental progressions are tools which originally arose from the work of Piaget (Hurst, n.d.). They are carefully designed detailed 'maps' which illustrate the increasingly sophisticated behaviours that a learner will display as they progress from being a novice to being an expert in any domain of learning (University of Melbourne, 2015).

To describe the developmental progression of collaborative problem solving (CPS) within the MeJ workshops, we adapted to our context the empirically validated developmental progression on one dimension developed within the ATC21S project (ATC21S Project, 2014).

The developmental progression incorporates both social (light orange colour in the table) and cognitive skills (blue colour in the table).

Level	Level description
6	The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem. The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.
5	The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem. The student promotes interaction and responds to peers' contribution but may not resolve differences.
4	The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases. The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses. The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.
3	The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases. He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher. The student reports his/ her activity.
2	The student attempts to better understand the problem through limited analysis; s/he explores two particular cases. He or she sets general goal, begins testing hypotheses. Interaction with peers is limited to brief acknowledgments on significant issues related to the research.
1	The student explores the problem space independently with no evidence of collaboration. His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case. No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.

Table 5. The developmental progression of the CPS competence within the MeJ context

Developmental progressions provide a tool for the teacher to be a systematic observer of student progress. They can help the teacher to notice the kinds of actions and behaviours that students demonstrate and to interpret these observations as evidence of cognitive or social growth. The developmental progression might be used as a frame of reference to interpret what they see the students doing, saying, making, or writing in collaborative problem solving activities. In this way, a teacher can assess students' level of development and identify the point of intervention where they are most likely to learn.

2.2. Written and oral communication skills of the research results

2.2.1 Definition of the competence

Within the math research workshops, **oral presentation of the math research results** implies the presentation in front of a large number of people, not necessarily math experts/ professionals, of the results, research approaches and/ or methods. The students have to adapt their discourse or presentation to the audience in order to make them understand the math concepts and procedures. Depending on the audience and the nature/ type of communication, the students might decide to use aids and/ or specific tools (computers, scale-models, etc.).

Writing the research article implies the use of both colloquial and scientific/ mathematical language for expressing ideas, for presenting the process and the knowledge gained. When writing the research article, the student will have to comply with rules and standards set by the scientific community.

This competence is lies on the borderline between mathematical competences and communication competences, and can be seen as a sub-skill of *communication in, with and about mathematics*. Mogens Niss and Tomas Hojgaard (Niss & Hojgaard, Competencies and Mathematical Learning - Ideas and inspiration for the development of mathematics teaching and learning in Denmark, 2011) define the *competence of communication in, with and about mathematics as*:

a. understanding of written, visual or oral texts produced by others, in different registers, about problems with a mathematical content;
b. written, oral or visual expression, at various levels of theoretical and technical precision, relative to such problems.

The written and oral communication skills are in part (b) of the definition. In order to present the research topic and results in a display space in the MeJ Congress, the students must adapt their discourse depending on the interlocutor, whether a student or a researcher. For this type of exercise, the students need to think in advance about the mathematical concepts used and the results obtained in the workshop and also to prepare for their presentation.

One might wonder why we think written and oral communication skills for the presentation of the research results are specific competences developed in the MeJ workshops – in other words, is the development of these written and oral presentation skills part of the research workshop? When a researcher publishes her work or presents them in a congress, she generally addresses a specialist audience and often she need not adapt her discourse (whether written or oral). However, audiences are not always made up of specialists and the ideas may prove of interest for non-specialists as well. In this case, it is recommended that the researcher explain her approach and findings in a less specialist language, which necessitates adapting the discourse to the interlocutors.

2.2.2. Make the skills visible

Lycée d'Altitude has a long experience in running MATh.en.JEANS workshops. Since 1998, the youth there have been doing mathematical research. At the end of the academic year, the students' outputs, articles and posters, are a way of showing their results and a good basis of recognition by the mathematicians'

community. The work invested in preparing the products contributes to the development of mathematical and (written) communication competences. Writing the research article requires synthesising the findings of all groups who researched the same topic, and good communication among the youth. The research article must be validated by an editorial board including university professors before it gets published on the website of the MATh.en.JEANS Association.

When we interview the alumni of the MeJ workshops, ease of oral expression (part of the communication competence) is quite often mentioned in their statements. The presentations in front of a diverse audience in the congresses, forums, seminars or in scientific events have contributed a lot to the development of their oral communication skills. The various oral presentations of the research topic and results force the students to confront their ideas and approaches, to adapt their discourse to their audience, who are not necessarily specialists in math. Thus, they are a source of positive change and enrichment.

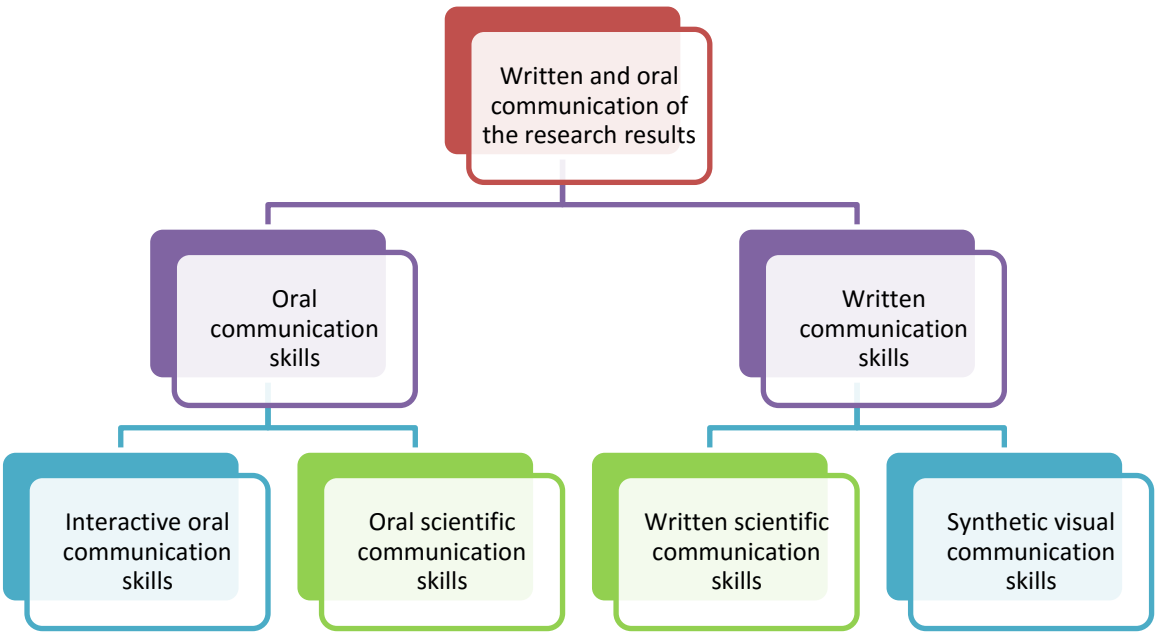


Figure 5. Diagram of competences and sub-skills pertaining to oral and written communication

We must bear in mind that there are two types of communication involved:

- group communication – when a group of students produce something together through which they communicate, in a synthesised, visual manner, the research topic and results (e.g. a poster) or when several groups of students who researched the same topic write a joint research report;
- individual communication – when a student presents the research topic and results in a conference, a scientific communication session, or in a competition, etc. Although the oral presentation is performed by the entire group, there is a period of time when an individual student presents on her own.

The behaviours described below take this into account.

Sub-skills	Behaviour	Behavioural indicator
Interactive oral communication (interactive presentation at a stand)	The students presents several aspects related to the research, at various levels of complexity (from the standpoint of mathematical knowledge) The student presents different examples and uses a variety of forms of support to facilitate understanding of the research topic. The student adapts the presentation in response to the audience's questions and areas of interest. The student engages the audience in activities related to the research presentation and urges the audience to ask questions. The student responds clearly and accurately to questions from the audience.	Has several versions of oral presentation of the research topic and results. Has prepared visuals to make the presentation interactive. Manages interaction with the audience and adapts the presentation to the audience's questions. Engages the audience – asks questions of the audience.
Oral scientific communication (presentation in a congress, seminar)	The student uses visual support (slideshow/ PPT) (accurate, key elements). The student speaks in public. The student present the research topic, the approach and the results. The student answers questions from the audience.	Uses visual support (slideshow, PPT etc.) Presents orally, in public, the research topic and results. Answers questions.
Synthetic visual communication (poster)	The group of students use IT programmes (software – word processing, image processing, Geogebra, etc.). The group of students synthesise the key ideas of the research topic and results. The group of students prepare a poster which is understandable by people who are not familiar with the research topic.	Uses IT programmes (word processing, image processing, Geogebra, etc.) to prepare the product. Synthesises the research topic and results.
Written scientific communication (research article)	Each member of the group contributes to the article about the research topic. The group of students write the article using appropriate It programmes. The group of students revise the content and the format of the article based on feedback from third parties (teachers, researcher, editor, etc.)	Writes a scientific article about the research topic and the results obtained. Structures the information about the approach used in research. Uses a computer to write the article.

Table 6. Sub-skills of the skill *oral and written communication of the research results* in MATh.en.JEANS workshops

2.2.3. Definition of criteria for each behavioural indicator

In order to define the quality criteria for each behavioural indicator, we chose to describe three levels of performance for each of the behavioural indicators.

Sub-skills	Behavioural indicator	Low (novice)	Medium (competent)	High (expert)
Interactive oral communication (interactive presentation at a stand)	Has several versions of oral presentation of the research topic and results.	Has only one version of the presentation, which s/he 'recites'.	Presents the research topic and a few particular cases/ examples.	Presents relevant and exciting parts of the research, adjusting them to the audience's interest.
	Has prepared visuals to make the presentation interactive.	Provides the visitors a handout with the general presentation of the topic/ results.	Uses various forms of support to present a few specific aspects of the research.	Uses most appropriate handouts/materials and convinces the audience to interact with them.
	Manages interaction with the audience and adapts the presentation to the audience's questions.	Avoids interaction with the audience, pretending not to hear questions.	Urges the audience to ask questions, listens to them, answers some of them, but does not adjust presentation as a response.	Invites the audience to ask questions, adapts the presentation in response to the questions.
	Engages the audience – asks questions of the audience.	Avoids addressing questions to the audience.	Asks simple questions of the audience. (e.g.: do you understand the research topic?)	Asks questions of the audience and adjusts presentation according to the answers received.
Oral scientific communication (presentation)	Presents orally, in public, the research topic and results.	Reads notes (from printout, from the slides) in front of the audience. Reads quickly about the research topic and/ or results.	Manages to look up from reading, but is hesitant in the presentation and/ or the presentation contains too many results which make it difficult to follow.	Presents freely, in public, the key elements of the research, has good diction and manages well the time allocated for the oral presentation.
	Uses visual support (slideshow, PPT etc.)	Does not use visual support.	Uses PPT/ slideshow, but it has too much text or visual effects that cause confusion.	Uses visual support (PPT/ slideshow) which is prepared properly, is clear and very well structured.
	Answers questions.	Avoids answering questions from the audience.	Replies to some questions from the audience, but the answers are rather unclear.	Provides clear and relevant answers to all the questions from the audience.

Sub-skills	Behavioural indicator	Low (novice)	Medium (competent)	High (expert)
Written scientific communication (research article)	Writes a scientific article about the research topic and the results obtained.	Prepare an article about the work done during the year, without including the scientific results or the approach.	Write a scientific article about the research, but the article does not include the contribution of all members of the group or the contribution of each is obvious (they do not use the same notations or style).	Prepare a scientific article on the topic and results of the research, which seamlessly incorporates the contribution of each group member.
	Structures the information about the approach used in research.	Prepare a poorly structured article.	Write about the research topic and results, but fail to present how their research progressed or the questions which they sought to answer – in order to structure the text.	Write about the research topic and results, provide the mathematical proof of the results and structure the article very well.
	Uses a computer to write the article.	Type the article on a computer with little attention paid to the look of the document.	Type the article on a computer in an orderly manner; uses the same type of character and same size font, indentations, spacing, etc.)	Type the article correctly (font, paragraphs, etc.) and insert mathematics formulae in the text.
Synthetic and visual communication (poster)	Uses IT programmes (word processing, image processing, Geogebra, etc.) to prepare the product.	Produce a poster without using IT instruments.	Produce a poster using IT instruments.	Produce a poster using a variety of IT programmes.
	Synthesises the research topic and results.	Produce a confusing, messy or overloaded poster.	Produce an attractive poster which presents the research topic and some of the results clearly.	Produce a very good quality poster (clear, attractive, etc.), validated by third parties, which includes all the important research results.

Table 7. Levels of performance for the sub-skills of written and oral communication skills

2.2.4. Developmental progression

The sub-skills of the *written and oral communication skills* are very different from one another – and obviously they depend on the type of communication (written or oral) and on the activity in the context of which they can be observed (presentation delivered at a stand/ in scientific events/ preparing a scientific article, etc.). Some students may not have the opportunity to develop all these sub-skills – for instance, if, during the academic year, they do not participate in any scientific event where they could present the research topic and results, then they will not have the opportunity to develop their oral scientific communication skills. For these reasons, we prefer to provide one developmental progression for each subskill separately.

The developmental progression of the interactive communication subskill

Interactive communication (delivered to visitors at a stand, such as in an exhibition/ science fair)	
Level	Description of the level
6	Upon sensing or finding out about the visitors' interests, the student grabs and holds their attention by presenting a captivating and highly relevant part of the research, in a perfectly adjusted language, using the most appropriate tools, engaging in a fluent dialog. *For example: involving young students in the analysis of some simple cases, skipping the mathematical proof for an 'unfamiliar with math' audience, pointing out when the audience's questions are similar to his/ her own questions related to the research.*
5	Upon sensing the visitors' interests, the student grabs their attention by presenting some of the research, in well-adjusted language, using appropriate tools, which s/he invites the audience to manipulate, play with, etc. He/ she asks and answers questions from the audience.
4	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience by inviting them to manipulate some materials, and addressing simple questions such as: Do you understand the research topic? What are your ideas about ….? What would be your answer to ….?
3	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience to a limited extent. The student uses materials prepared before the interactive communication takes place. *For example, the student gives examples, provides a schema, plays games with people from the audience, models/ simulates in order to facilitate the audience's understanding of the research topic.*
2	The student uses a few materials (paper, computer, etc.) to illustrate some of the points made in the presentation, paying no attention to the visitors' potential interests; fails to engage the visitors. *For example, the student draws a schema on a sheet of paper, uses a computer for presenting some configurations/ collected data/ etc., uses a board game).*
1	Paying no attention to the visitors' potential interest, the student delivers an unadjusted presentation of the research topic, and hands visitors a printed outline, failing to engage them.

Table 8. The developmental progression of the interactive communication subskill within MeJ

The developmental progression of the oral scientific communication subskill

Oral scientific communication (in a plenary)	
Level	Description of the level
6	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student provides clear and relevant answers to all the questions from the audience.
5	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student answers questions from the audience mostly clearly.
4	The student delivers an oral presentation of his/ her research work using a clear well-structured visual support (e.g. a slideshow/ PPT). The student answers some questions, but his/ her answers tend to lack clarity.
3	The student delivers an oral presentation of his/ her research work, using visual support (e.g. a slideshow/ PPT). However, the PowerPoint presentation has too much text and/ or confusing visual effects. He/ she makes the presentation sometimes simply reading the text from the slide(s), and refraining from answering questions.
2	The student delivers a simple oral presentation of his/ her research work, without relying on reading, failing to use visual support to facilitate the understanding of the presentation, and refraining from answering questions.
1	The student presents his/ her research work orally, by simply reading out his/ her notes in a rather dull manner, and refraining from answering questions.

Table 9. The developmental progression of the oral scientific communication subskill within MeJ

Image 1. Oral presentation of the research results at the Lyon Congress (2016)

The developmental progression of the written scientific communication subskill

Written scientific communication (research article)	
Level	Description of the level
6	Seamlessly integrating each members' contribution, the group of students has produced a well-structured, well written and neatly typed research article, which presents the research topic, the approach, the results & the mathematical proof(s) in a highly appropriate language.
5	Incorporating the contribution of each member of the team, the group of students has produced a well-structured, neatly typed research article which presents the research topic, the approach, the results & the mathematical proof(s), in need of limited editing.
4	Some members of the group of students have contributed to producing a neatly typed, quite well-structured and mostly appropriately worded research article, which contains the research topic, their research approach, the research results & the mathematical proof(s).
3	Some members of the group of students or a member of the group has produced a complete although poorly structured research article, typed in a rather messy manner, in a register that fails to comply with the standards of a research article.
2	Some members of the group of students or a member of the group has typed up an incomplete and poorly structured article, which presents the topic and the research that was carried out, but fails to present the scientific results, etc.
1	Some members of the group of students or a member of the group has typed up a simplistic, partial report on the research they have carried out, failing to specify essential information e.g. the topic and/or the approach and/ or results, etc.

Table 10. The developmental progression of the written scientific communication subskill within MeJ

The developmental progression of the synthetic and visual communication subskill

Synthetic and visual communication (poster)	
Level	Description of the level
6	The group has produced and made available online a very clear and attractive item of visual communication (poster in pdf format, very well edited video recording), which shares the research topic, as well as all the major results in an original manner, using highly appropriate computer software/ tools to thoroughly engage viewers. .
5	The group has produced a very clear and attractive item of visual communication which shares the research topic as well as all the major results, using computer software/ tools to show the most relevant parts of it.
4	The group has produced a clear and attractive item of visual communication which shares the research topic as well as most of the results using computer software/ tools to show some parts of it.
3	The group has produced an attractive although not thoroughly clear item of visual communication of the research topic and some results using computer software/ tools to show some parts of it.
2	The group has produced a simple item of visual communication of the research topic and some results, which however tends to be mostly confusing and/or overloaded, blurring the key points to get across.
1	The group of students has produced a simplistic item of visual communication of the research topic (a poster), which fails to attract viewers' interest.

Table 11. The developmental progression of the synthetic and visual communication subskill within MeJ

2.3. Competence: Making use of aids and tools (IT included)

2.3.1. Definition of the competence

This competence has been defined by Mogens Niss and Tomas Hojgaard (Niss M. &., 2011) as consisting of:

a. having knowledge of the existence and properties of the diverse forms of relevant tools used in mathematics and having an insight into their possibilities and limitations in all sorts of contexts;
b. being able to reflectively use such aids.

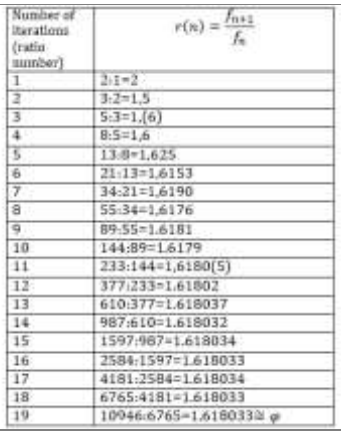
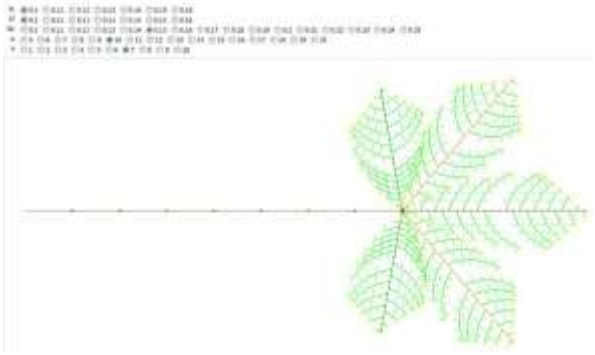
OECD considers *making use of tools* a capability (OECD, 2013). It is defined as follows: "this ability involves knowing about and being able to make use of various tools that may assist mathematical activity, and knowing about the limitations of such tools." Actually, the OECD definition is quite similar with the definition provided by Niss and Hojgaard; *reflection* makes the difference, as the Danish authors mention the *reflective* use of aids and tools.

To define this competence, it is necessary to first define aids and tools. According to Cambridge Dictionaries (Cambridge University, 2015) *a tool is a piece of equipment that you use with your hands to make or repair something* while *an aid is a piece of equipment that helps you to do something*.

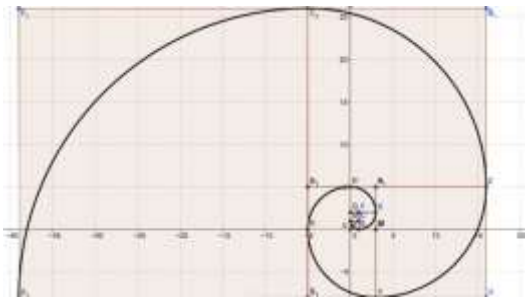
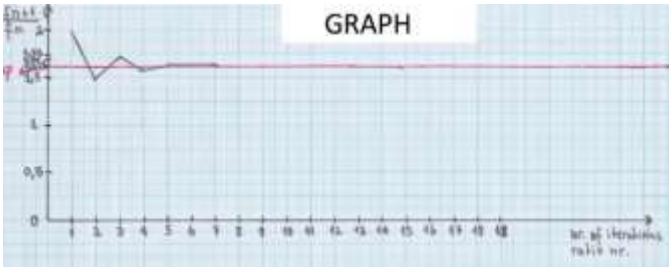
When doing math, people made use of diverse technical aids *both to represent and maintain mathematical entities and phenomena, and to deal with them, e.g. in relation to measurements and calculations* (Niss M. &., 2011). There are a lot of examples which show the use of aids in mathematics; e.g. use of manipulatives which provide a way for children to learn concepts and experiment in a developmentally appropriate way. Different materials (LEGO-bricks, blocks, geometric templates, etc.) support conceptualisation, study of patterns and connections. Calculators and computers, different graphic programmes (e.g. Geogebra), spreadsheets, etc. are also tools used when doing mathematics – e.g. portraying a mathematical relationship, finding exact or approximate solutions, etc.

What kind of aids and tools can/ do students use in the In the MATh.en.JEANS workshop? During our work within the MeJ workshops we noticed different aids and tools the students used. Here are some examples of aids and tools our students used when researching the *Modelling plant growth* topic.

	Types of aids and tools	**Reasons for using the aids and tools**
Sycamore tree leaf		observation/ recognising mathematical structures
		analysis of the midrib and the veins/ portraying mathematical relationships

Types of aids and tools		Reasons for using the aids and tools
Protractor		identifying the measure of the angle between the midrib and the veins/ portraying mathematical relationships
Calculator		calculating values ($r(n) = \frac{f_{n+1}}{f_n}$) to identify patterns/ portray mathematical relationships
Mathematica – software	 	creating a model to check the solution/ ascertain the reasonableness of a mathematical solution and any limits and constraints to that solution, given the context of the problem

Number of iterations (ratio number)	$r(n) = \frac{f_{n+1}}{f_n}$
1	2:1=2
2	3:2=1,5
3	5:3=1,(6)
4	8:5=1,6
5	13:8=1,625
6	21:13=1,6153
7	34:21=1,6190
8	55:34=1,6176
9	89:55=1,6181
10	144:89=1,6179
11	233:144=1,6180(5)
12	377:233=1,61802
13	610:377=1,618037
14	987:610=1,618032
15	1597:987=1,618034
16	2584:1597=1,618033
17	4181:2584=1,618034
18	6765:4181=1,618033
19	10946:6765=1,618033≅ φ

Types of aids and tools		Reasons for using the aids and tools
Geogebra		representing values ($r(n) = \frac{f_{n+1}}{f_n}$) to identify patterns/ assist in implementing processes and procedures to determine mathematical solutions
Plotting paper		representing graphs/ assisting in implementing processes and procedures to determine mathematical solutions

Python programme	`import turtle`
	`>>>def _fibonacci(n):`
	`  fib1 = 0; fib2 = 1`
	`  if n==0:`
	`  return fib1`
	`  elif n == 1:`
	`  return fib2`
	`  else:`
	`  fori in range(n-1):`
	`  fib = fib1+fib2`
	`  fib1, fib2=fib2, fib`
	`  return fib`
	
	`>>>def _square(t, seg, size, fib):`
	`  fori in range(seg):`
	`  t.fd(size)`
	`  t.left(90)`
	`  t.write(fib)# write the current Fibonacci next to the square`
	`  t.fd(size)`
	
	`>>>defdraw_spiral(size=10):`
	`  t = turtle.Turtle()`
	`  seg = 3 # set to 5 segments after first pass`
	`  fori in range(size):`
	`  fib = _fibonacci(i)`
	`  if (fib>0):`
	`  size = fib * 10 # a multiplier to give squares a reasonable size`
	`  _square(t, seg, size, fib):`
	`  seg = 5`
	`  turtle.done()`
	
	`>>>draw_spiral()`
		representing mathematical concepts (building the Fibonacci spiral using squares)/ visualisation

Table 12. Aids and tools students used when working on the topic *Modelling plant growth*

Some other non-conventional aids and tools used by the students in the MeJ workshop in 2014 – 2015 are:

- straw models of the bicycle wheels (Research topic: The bicycle wheel)

Image 2. Bicycle wheels made of straw

- plywood model of a shaped serrated road and bicycle wheel (Research topic: *The bicycle wheel*)

Image 3. Plywood model of a shaped serrated road and bicycle wheel

- mock-ups (Research topic: *The paradox of Braess*)

Image 4: Mock-up

These aids and tools involve different types of mathematical representation. That's why the competence *making use of aids and tools* is closely linked to the competence *representing mathematical entities (objects and situations)*. According to Niss (Niss M. &., 2011), the competence of *representing mathematical entities competence* comprises:

- understanding and utilising (decoding, interpreting, distinguishing between) different sorts of representations of mathematical objects, phenomena and situations;
- understanding and utilising the relations between different representations of the same entity, including knowing about their relative strengths and limitations;
- choosing and switching between representations.

The competence *making use of aids and tools* is also linked also with the competence of *communicating in, with and about mathematics*, as tools can also have an important role in describing the research topic and communicating results.

Remark:

In the context of the Romanian high-school math curriculum, "mathematical tools" refer to mathematical procedures (such as algorithms) and mathematical concepts (Ministerul Educației, Cercetării și Tineretului - CNC, 2004). However, for the purposes of assessing of the students' competences developed within the MeJ workshops, "mathematical aids and tools" refers only to the physical and digital tools described in this section.

2.3.2. Make the competence visible

The competence *making use of aids and tools* is present across the three mathematical processes the students go through when working on the MeJ research tasks. The three mathematical processes are identified by OECD (OECD, 2013) who describe what individuals do to connect the context of the math research task with mathematics and thus find solutions to the research task. The three mathematical processes are:

- formulating situations mathematically: the students recognise and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualized form (OECD, 2013). In the MeJ context, to tackle the research topic, the students design a mathematical structure, representations and specificity.
- employing mathematical concepts, facts, procedures and reasoning: the students apply mathematical concepts, facts, procedures, and reasoning to solve mathematically-formulated problems to obtain mathematical conclusions (OECD, 2013). In the MeJ context, the students work on a model of the research task, establish regularities, identify connections between mathematical entities, and create mathematical arguments.
- interpreting, applying and evaluating mathematical outcomes: the students reflect upon mathematical solutions, results, or conclusions and interpret them in the context of the problem (OECD, 2013). In the MeJ context, the students construct and communicate explanations and arguments in the context of the research task, reflect on both the modelling process and its results.

The way in which the competence *making use of aids and tools* reveals itself within the three processes is described in table 2, which is an adaptation of figure proposed by OECD (OECD, 2013) showing the relationship between mathematical processes and fundamental mathematical capabilities.

Formulating situations mathematically	Employing mathematical concepts, facts, procedures and reasoning	Interpreting, applying and evaluating mathematical outcomes
Use aids and tools in order to recognise mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various aids and tools that may assist in implementing processes and procedures for determining mathematical solutions	Use aids and tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the research task

Table 13. Relationship between the mathematical processes and the competence *making use of aids and tools*

After observing the aids and tools students use while researching the MeJ math topics and analysing the definition of the competence *making use of aids and tools*, we define the behavioural indicators of this competence.

Components	Behaviour	Behavioural indicator
Having knowledge of the existence and properties of the diverse forms of relevant tools used in mathematics and having an insight into their possibilities and limitations in all sorts of contexts	Student • chooses the appropriate tools and aids s/he already used during the math lessons by describing their possibilities and limitations in the research topic context; • searches for appropriate tools and aids s/he might use when researching the topic, learns how to use them, and describes their possibilities and limitations in the research topic context; • constructs the tools and aids necessary for understanding and solving the research task. They represent research topic related data and/ or relationships by using tools and aids. They make appropriate use of various aids and tools that may assist in implementing processes and procedures for determining mathematical solutions. They interpret the resulting findings from using the tools and aids.	Identifies and uses aids and tools that may assist mathematical research activity and clearly describes the limitations of the used tools in the research topic context.
Being able to reflectively use such aids	Student analyses the use of aids and tools (which aids and tools they have used, why they used these aids and tools). They evaluate the use of the aids and tools and communicate their learning from this experience. They may reflect on what else they could have used, and why they ended up not using those aids and tools.	Reflects meaningfully on the use of aids and tools.

Table 14. Framework of the making use of aids and tools competence within the MeJ context

2.3.3. Definition of criteria for each behavioural indicator

In order to define the quality criteria for each behavioural indicator, we chose to describe three levels of performance for each of the behavioural indicators. In describing these levels, we took into consideration the observations made during the working process within the MATh.en.JEANS workshop and the reviewed literature: (OECD, 2013) and (Niss, 2011). We work with two dimensions of a person's mastery of the competency: the degree of coverage – used to indicate the extent to which the person masters those aspects which characterise the competency, and the technical level – used to indicate how conceptually and technically advanced the entities and tools are with which the person can activate the competence (Niss, 2011).

Behavioural indicator	Low (novice)	Medium (competent)	High (expert)
Identifies and uses aids and tools that may assist mathematical research activity and clearly describes the limitations of the used tools in the research topic context.	Identifies and uses aids and tools s/he already used during the math lessons; the used aids and tools may assist in formulating the research task mathematically. Describes, in general terms, the possibilities and the limitations of the used tools.	Searches for appropriate tools and aids s/he might use when researching the topic, and, if needed, learns how to use them. Uses aids and tools which may assist in implementing processes and procedures for determining mathematical solutions. Using the tools involves a sequence of process or linking different information. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means.	Identifies and uses complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures to determine mathematical solutions; the aids and tools are either tools s/he already used during the math lessons or new tools s/he identifies. If necessary, constructs new tool(s) necessary for understanding and solving the research task. Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool.
Reflects meaningfully on the use of aids and tools.	Analyses the use of aids and tools (which aids and tools they have used and why).	Evaluates the use of the aids and tools and communicates their learning from this experience.	Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used, and why s/he ended up not using those aids and tools.

Table 15. Criteria of behavioural indicators of the competence *making use of aids and tools* within the MeJ context

2.3.4. Developmental progression

The developmental progression is a tool which dynamically describes how the respective student develops the competence over time. This tool actually promotes the development of the competence *making use of aids and tools* within MATh.en.JEANS. The developmental progression is a 'map' which illustrates the growth of the competency. It supports the MeJ facilitator (teacher) in carefully observing his/ her students and identifying areas where there is room for improvement.

Level	Level description
6	The student identifies and uses familiar or non-familiar, complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures for determining mathematical solutions. Constructs new tool(s) - necessary for understanding and solving the research task. Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool. Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used, and why s/he ended up not using those aids and tools.
5	The student identifies and uses familiar or non-familiar, complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures for determining mathematical solutions. Has the idea to create a new tool or aid necessary for understanding and solving the research task (e.g. a computer application). Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool. Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used.
4	The student identifies and uses familiar or non-familiar aids and tools for implementing processes and procedures for determining mathematical solutions. He learns how to use the non-familiar aids and tools. Using the tools involves a sequence of process or linking different information. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means. Evaluates the use of the aids and tools and communicates their learning from this experience.
3	The student identifies and uses familiar aids and tools. Using the tool involves a sequence of process or linking different information. The aids and tools used may assist in implementing processes and procedures to determine mathematical solutions. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means. Evaluates the use of the aids and tools without sharing their learning from this experience.
2	The student identifies and uses tools and aids s/he already used during math lessons for formulating the research task mathematically. Describes, in general terms, the possibilities and the limitations of the used tools and aids. Analyses the use of both aids and tools (which aids and tools they have used, why they used these aids and tools).
1	The student identifies and uses familiar tools and aids (e.g. measuring instruments, objects that are mentioned in the text of the research task). S/he describes, in general terms, the possibilities and the limitations of the used tools. S/he uses the tools and aids in order to recognise mathematical structures in the text of the research topic. S/he names the used tools and formulates one reason for using the identified tools.

Table 16. The developmental progression of the competence making use of aids and tools within MeJ

3. Methods and instruments for assessing students' competence developed through math research

For the development of the assessment methodology (methods and instruments) for each competence, we bear in mind on the one hand the aspects listed in subchapter 1.2 How we view assessment in the mathematical research workshops and, on the other hand, the developmental progression for each competence.

As we aim for formative assessment, the methodology should:

- be based on the developmental progression, which describes in a dynamic manner the way in which the student progresses in developing a particular competence in time, and which supports the teacher and the student in identifying where the student is at a given time, and what she should do to reach the next level. Understanding of the developmental progression by the student helps her figure out how she learns and become aware of the process she goes through as she develops her competence. As for the teacher, understanding the progression helps her in the assessment process and in better supporting the students' learning.
- Involve both the teacher and the student – the two partners in the learning process.

Consequently, in the competence assessment methodology we include as assessment methods self-assessment (by the student) and observation (by the teacher); the self-assessment sheet and the teacher's observation sheet are instruments which include the description of progression in the development of the assessed competence. The student's self-assessment sheet needs to be discussed with the students at the beginning of the year, in the first meeting, prior to announcing the research topics. At the same time, or one week prior to using the sheet at the latest, the students should practice filling in this document under the guidance of the teacher: the teacher fills the sheet in front of the students, and thinks aloud, after which the students fill it in individually and ask clarification questions, if needed. Thus, the teacher prepares the students for self-assessment.

As the students' research activity is sometimes carried out outside the classroom, so the teacher cannot constantly observe each student who participates in the research workshop, in order to fill in the observation sheet it is necessary for the teacher to analyse the students' research diaries/ report sheets, etc. The research diary is a combined notebook and learning diary; in addition to the students' notes, tasks, deadlines, ideas how to approach the topic/ solve the problem, it includes the students' reflections on the activities in the workshop and on how the research progresses (what she has learned about herself and about her colleagues, what new knowledge she has mastered while working on the research topic). The MUAT reporting form is described in subchapter 3.3. Both provide the students with an opportunity to reflect on their learning process: on the one hand, they are useful instruments for developing learning to learn skills, and the other hand, they are useful sources of information for conducting assessment.

In the assessment methodology, in addition to the assessment methods and tools, we also include reporting sheets. They synthesise the results of the assessment and support the teacher in identifying

what progress the student has made in the development of a competence over a longer period of time (one year or more).

3.1. Methods and instruments for assessing students' collaborative problem solving competence

For assessing students' collaborative problem solving competence developed within the MeJ workshops, we suggest the following methods and instruments:

Methods	Instruments
Observation, analysis of the information and reflections in the students' logbooks	The CPS observation sheet (to be filled out 2 times per school year)
Self-assessment (2 times per school year)	The CPS self-assessment sheet

Table 17. Methods and instruments for assessing students' collaborative problem solving

The students' logbooks are useful only for identifying aspects mentioned in the observation sheet which the teacher didn't/ couldn't observe during the MeJ meetings or for checking the accuracy of the observed actions and behaviour. If there is no concordance between the teacher's observations and the student logbook, the teacher might decide to take a closer look at the specific discordant issues.

3.1.1 How to use the observation sheet?

Filling in the observation sheet

In order to fill in the observation sheet, the teacher continuously observes the student's activity and makes a note of the actions/ behaviour observed. The teacher fills in the CPS observation sheet twice in the academic year. We recommend that the time for this to be done should be December and May. In order to fill in the observation form, the teacher colours the activities and behaviours observed at a particular student. Obviously, as the evaluation instrument is a description of the developmental progression of a competence, an action (or behaviour) may appear on several levels, the difference consisting in the manner in which something is done; for instance, if *the student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases* (level 3 in the competence's developmental progression), then she *has explored the problem space and analysed a particular case* (level 1) and *has attempted to better understand the problem through limited analysis; s/he explores two particular cases* (level 2). On the other hand, if *the student starts to share resources and information with colleagues – the shared resources and information are sometimes irrelevant* (level 3), then prior to this, in the same academic year, the teacher may not have observed the student *sharing information or resources* (level 1).

Interpretation of information in the observation sheet (completed)

The highest level of the developmental progression where almost all the activities or behaviour are marked will be considered the student's level in the development of the collaborative problem solving

competence, while the level on the progression where about half the activities or behaviour are marked is the student's zone of proximal development. As the observation sheet includes both cognitive and social skills, it is possible that a student is at one level in the development of her cognitive skills, and at another level in the development of her social skills (see image 5).

The CPS observation sheet

Use the observation sheet at the beginning of December and at the end of May of each school year. Mark with highlighters (use different colours for the two dates) to identify the kinds of activities and behaviours that the student demonstrates.

Level	Student Name: XXXXXXXXXXXXXXXXXXXXX	Date 1: December 1, 2015 Date 2:
6	The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem. The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.	
5	The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem. The student promotes interaction and responds to peers' contribution but may not resolve differences.	
4	The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases. The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses. The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.	
3	The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases. He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher. The student reports his/ her activity.	
2	The student attempts to better understand the problem through limited analysis; s/he explores two particular cases. He or she sets general goal, begins testing hypotheses. Interaction with peers is limited to brief acknowledgments on significant issues related to the research.	
1	The student explores the problem space independently with no evidence of collaboration. His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case. No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.	

Image 5. Example of a completed CPS observation sheet

In this case, the student's cognitive skills are at level 2, and her instructional level or zone of proximal development is at level 3, while her social competences are at level 4, and her zone of proximal development is level 5.

3.1.2. How to use the self-assessment sheet?

Preparing students for using the self-assessment sheet

The students' self-assessment sheet must be discussed with those involved in the math research workshops at the beginning of the academic year. They must understand the text of the sheet, which is why the presentation of the actions and behaviour in the self-assessment sheet and/ or provision of relevant examples are absolutely necessary. Before self-assessment, the student need to practice this approach, under the teacher's guidance, meaning that the teacher fills in the sheet, thinking aloud in front of the students, after which the students practice individually how to fill in the sheet and ask questions if anything is unclear.

Filling in the self-assessment sheet

The students will fill in the self-assessment sheet at the same times when the teacher does (December/ January and May during the academic year). Filling in the sheet requires two actions: marking with a particular colour the activities and behaviour which the student thinks she has accomplished, and explaining why she marked those particular activities and behaviour (what exactly has made her think that the marked actions and behaviour have been displayed). The explanation of the choice of actions and behaviour in the self-assessment sheet is a good opportunity to reflect on the research activities carried out and on the progress in developing a particular competence. After filling in the sheet, the student should hand in a copy to the teacher.

Interpreting the information in the self-assessment sheet (completed)

Interpreting the information in the self-assessment sheet is done similarly to the interpretation of the information in the observation sheet.

3.1.3. Assessment result

To record their assessment of the students' collaborative problem solving competence, teachers will use reporting sheet 1. For instance, in image 6, the reporting sheet shows that the student demonstrates level 4 in the development of the competence – according to the teacher's assessment, and level 5 in the development of the competence – according to the student's self-assessment.

Reporting sheet 1

Student name: XXXXXXXXXXXXXXXXXXXX Date: December 2015

Level	Level description	Teacher assessment	Student assessment
6	The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem. The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.		
5	The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem. The student promotes interaction and responds to peers' contribution but may not resolve differences.		
4	The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases. The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses. The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.		
3	The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases. He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher. The student reports his/ her activity.		
2	The student attempts to better understand the problem through limited analysis; s/he explores two particular cases. He or she sets general goal, begins testing hypotheses. Interaction with peers is limited to brief acknowledgments on significant issues related to the research.		
1	The student explores the problem space independently with no evidence of collaboration. His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case. No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.		

Image 6. Example of a completed reporting sheet

If the teacher is interested in having the assessment results on both dimensions (cognitive and social) of the collaborative problem solving competence, they might corroborate the highlighted statements from

the CPS observation sheet with the colour code used in the CPS developmental progression table (see table 5) and use reporting sheet 2. Both reporting sheets are adaptations of the Learning readiness reports (University of Melbourne, 2015). An analysis of the especially significant differences between the results of the teacher's observations and student's self-assessment is recommended.

The student needs to know the result of the assessment (the current level of competence development and the zone of proximal development) in order to be able to identify what she needs to do to further develop the competence. The teacher must identify appropriate ways to support the student in further developing her competence.

Comparisons between the two assessments (at the beginning of December and at the end of May) will inform the development of each student's CPS competence within the MeJ workshop during one academic year.

3.1.3. The CPS observation sheet

Use the observation sheet at the beginning of December and at the end of May of each school year. Mark with highlighters (use different colours for the two dates) to identify the kinds of activities and behaviours that the student demonstrates.

Level	Student Name:	Date 1: Date 2:
6	The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem. The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.	
5	The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem. The student promotes interaction and responds to peers' contribution but may not resolve differences.	
4	The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases. The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses. The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.	
3	The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases. He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher. The student reports his/ her activity.	
2	The student attempts to better understand the problem through limited analysis; s/he explores two particular cases. He or she sets general goal, begins testing hypotheses. Interaction with peers is limited to brief acknowledgments on significant issues related to the research.	
1	The student explores the problem space independently with no evidence of collaboration. His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case. No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.	

3.1.4. The CPS self-assessment sheet

Use the self-assessment sheet at the beginning of January and at the end of May of each school year. Mark with highlighters (different colours for the two dates) to identify the kinds of activities and behaviours you think you demonstrate. Then, fill out the second column in the table.

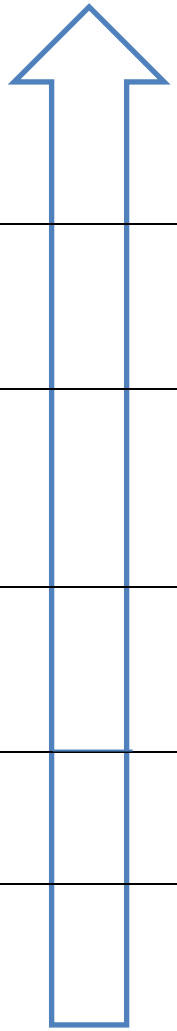
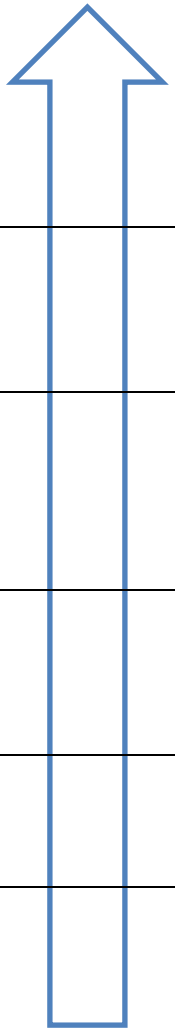
Student Name:	Date 1: Date 2:	Explain your reasons for marking the statements. Give concrete examples of what you have done for supporting your choice.
I work with others through the problem solving process and assume group responsibility for the success of our research tasks. I work together with my peers through the problem efficiently and systematically using only relevant resources. We plan(-ned) the research strategy based on a generalised understanding of cause and effect, and reorganize(-d) our shared understanding of the problem. I tailor communication with my peers, incorporate their contributions and feedback and resolve conflicts.		
I identify the necessary sequence of subtasks; I explore(-d) more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. My actions are planned and purposeful, identifying cause and effect and basing my/ our goals on prior knowledge. I reconstruct(-ed) understanding of the problem. I promote interaction in our group and respond to my peers' contribution but may not always resolve differences.		
Together with my peers, we divided the research task into subtasks, and I worked hard to successfully complete subtasks/ simpler tasks. I explore(-d) four particular cases. I identify(-ied) connections and/ or patterns in approaching the research task; I identify(-ied) sequences of cause and effect, I modify(-ied) hypotheses. I am aware of my own and my peers' abilities. We reach(-ed) a common understanding of our research topic. We started to plan strategies for finding solutions and refined our goals.		
I put some effort into finding solutions to the research task by stating the research topic in my own words, exploring three particular cases. I started to share resources and information with my peers, but sometimes the shared resources		

and information were not relevant for our research. I sometimes asked for support from the teacher. I report(-ed) my activity on the research task to my group.	
I attempt(-ed) to better understand the research topic by reading the text of the problem aloud several times, I explore(-d) two particular cases. I'm aware that we have to complete the research task, I started testing hypotheses. In the interaction with my peers, I acknowledge communication on significant issues related to the research.	
I explore the problem space independently (work alone at home). My participation in the discussions during the weekly meetings is very little. I don't really understand the research topic. I focus on isolated pieces of information, I explore(-d) one particular case. I missed more than three weekly meetings, interaction with my peers is limited to brief acknowledgments without providing information or resources.	

3.1.5. Reporting sheets

Reporting sheet 1

Student's name: _____ Date: _____

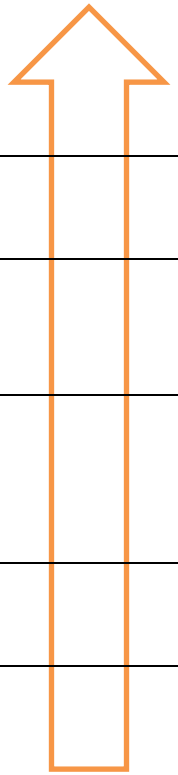
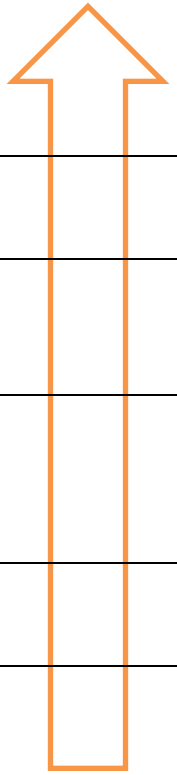
Level	Level description	Teacher assessment	Student assessment
6	The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem. The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.		
5	The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem. The student promotes interaction and responds to peers' contribution but may not resolve differences.		
4	The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases. The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses. The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.		
3	The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases. He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher. The student reports his/ her activity.		
2	The student attempts to better understand the problem through limited analysis; s/he explores two particular cases. He or she sets general goal, begins testing hypotheses. Interaction with peers is limited to brief acknowledgments on significant issues related to the research.		
1	The student explores the problem space independently with no evidence of collaboration. His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case. No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.		

Reporting sheet 2
Student's name: _____ Date: _____

Cognitive dimension of CPS

Level	Level description	Teacher assessment	Student assessment
6	The student works through the problem efficiently and systematically using only relevant resources. He/ she plans the research strategy based on a generalised understanding of cause and effect, and reorganizes understanding of the problem.		
5	The student identifies the necessary sequence of subtasks; explores more than four particular cases by using ICT (e.g. Geogebra) and/ or manipulatives. The student's actions are planned and purposeful, identifying cause and effect and basing their goals on prior knowledge. He/ she reconstructs the understanding of the problem.		
4	The student divides the research task into subtasks, perseveres to successfully complete subtasks and simpler tasks, and explores four particular cases. The student identifies connections and/ or patterns in approaching the research task; he/ she identifies sequences of cause and effect, and modifies hypotheses.		
3	The student demonstrates effort towards finding solutions to the research task by stating the research topic in his/ her own words, and exploring three particular cases.		
2	The student attempts to better understand the problem through limited analysis; s/he explores two particular cases. He or she sets general goal, begins testing hypotheses.		
1	The student explores the problem space independently with no evidence of collaboration. His/ her approach is unsystematic and focusing on isolated pieces of information; s/he explores one particular case.		

Social dimension of CPS

Level	Level description	Teacher assessment	Student assessment
6	The student works collaboratively through the problem solving process and assumes group responsibility for the success of the research tasks. The student tailors communication, incorporates contributions and feedback from peers and resolves conflicts.		
5	The student promotes interaction and responds to peers' contribution but may not resolve differences.		
4	The student is aware of his/ her and the peers' abilities. They reach a common understanding, start to plan strategies for finding solutions and refine goals with their peers.		
3	He/ she begins to share resources and information with the peers – resources and information shared are sometimes not relevant. The student sometimes asks for support from the teacher. The student reports his/ her activity.		
2	Interaction with peers is limited to brief acknowledgments on significant issues related to the research.		
1	No evidence of participation, interaction with peers is limited to brief acknowledgments without providing information or resources.		

3.2. Methods and instruments for the assessment of the written and oral skills for communication about the research results

The assessment of these skills is done when communication actions take place and when a research poster/ article is produced. Some students may not participate in congresses/ students' communication sessions, and then there is no way to assess their oral scientific communication skills. We consider that the students' participation in the project forum organised at the school, writing of the scientific article and producing the poster to share the research results should be activities included in the math research workshops. For this reason, it should be possible to assess the interactive oral communication skills, the synthetic and visual communication skills, and the written scientific communication skills of all students who participate in the MeJ workshops.

As writing a research article and producing a poster to share the research results occur only once in the course of an academic year, the assessment of the students' progress in developing the written scientific communication skills cannot be done if the students take part in the workshops only one year. The situation is similar for oral communication skills as well, if the students participate in only one event where they make an oral presentation.

Assessment of written and oral scientific communication skills cannot leave out the assessment of the communication product. In the following, we will use the word "product" for any interactive oral presentation, scientific presentation, research article, or poster sharing the research results.

For assessing the students' oral presentation and writing skills (related to the presentation of the research results), we suggest the following methods and instruments:

Methods	Instruments
Observation, product analysis (as often as the student/ group of students produces a product to communicate the research results) and analysis of the research diary	Observation sheet (one sheet for each sub-skill)
Self-assessment (as often as the student/ group of students produces a product that communicates the research results)	Self-assessment sheet (one sheet for each sub-skill)
Peer-assessment (as often as the student/ group of students produces a product that communicates the research results)	Observation sheet (one sheet for each sub-skill)
External assessment (as often as the student/ group of students produces a product that communicates orally the research results)	Assessment questionnaire (one questionnaire for each sub-skill of the oral communication competence)

Table 18. Methods and instruments for the assessment of communication skills related to the presentation of research results

3.2.1. How to use the observation sheet?

Filling in the observation sheet

The observation sheet is filled in by the teacher to collect information needed for the assessment of the written and oral communication skills. The same sheet is used by the students' peers for assessing their oral communication skills.

In order to fill in the sheet, the teacher must observe the activities where the products are prepared; this activity is very important because it is only the teacher and the student herself (or the group) who can have information about the process of preparing the product. The research diary may also provide information about the process – this is why analysis of the diary provides the teacher with one more source of information, especially because some activities related to the product preparation take place outside the weekly workshops. In the observation sheet there are statements relative to the product – therefore, product analysis is also useful prior to filling in the sheet.

Peer assessment is recommended to be done by 1-2 colleagues of the student who participate in the research workshops, attend the event, but are not involved in the presentation of their own research results in the same time period. As they participated in the workshop, they know the observation sheet and can fill it without needing additional explanation. The students should fill in the observation sheet right after the interactive oral presentation/ scientific presentation and hand it over to the teacher.

The teacher fills in the adequate observation sheet right after the communication of the research results. In order to fill in the observation form, the teacher marks the activities and behaviour observed at a student in a certain colour. As the assessment instrument contains the developmental progression of the sub-skill, an action (or behaviour) may appear at different levels, the difference being in the manner in which they are performed. For instance: if the student *asks questions from the visitors and responds to the questions of the audience* (interactive oral communication – level 5), then it is likely that she can also *ask simple questions* (interactive oral communication – level 4).

Interpreting the information in the observation sheet (completed)

The highest level reached, where all (or almost all) the activities and behaviour are marked will be considered the level of competence (for the respective domain), while the level where approximately half of the activities and behaviour are marked is the respective student's zone of proximal development.

3.2.2. How to use the self-assessment sheet?

Preparing the students for filling in the self-assessment sheet

The students' self-assessment sheet must be discussed with the students involved in the research workshop at the beginning of the academic year. They must understand the text of the sheet – which is why it is absolutely necessary for the teacher to present the actions and behaviours in the form and provide examples. If the students have already filled in self-assessment sheets for another competence (e.g. the CPS self-assessment sheet), we think it is not necessary to do guided practice for this self-assessment sheet, because the manner of filling it in is similar.

Filling in the self-assessment sheet

The student will fill in the self-assessment sheet immediately after the communication of the results of their research. Like with the CPS self-assessment sheet, in this case also filling in the sheet requires two actions: marking with a particular colour the activities and behaviour that the student considers she has displayed, and explaining the reason why she has marked those specific levels activities and behaviour (what specifically makes her think that she has accomplished the actions or displayed the behaviour she has marked). After filling it in, the student hands over a copy of the self-assessment sheet to the teacher.

Interpreting the information in the self-assessment sheet (completed)

Interpreting the information in the self-assessment sheet is done in a similar manner to interpreting the information in the observation sheet.

3.2.3. How to use the assessment questionnaire?

Filling in the assessment questionnaire

The assessment questionnaires are filled in by members of the audience (external to the research workshop attended by the student), who accept, upon the teacher's request, to fill in the questionnaire after having attended the presentation or participated in the interactive oral presentation performed by the student.

After having them filled in, the teacher collects the questionnaires. We recommend that the teacher show the student the filled in questionnaires: on the one hand, it is good for the student to be aware of the manner in which the audience has perceived the oral interactive presentation / scientific presentation, and on the other hand, the advice provided for the improvement of the presentation can be helpful in the learning process.

Interpretation of the information gathered via the questionnaire

For the information gathered via the questionnaire to be useful in the assessment of the level of communication skills development, it is necessary to correlate the responses in the questionnaire with the developmental progression levels for the development of the relevant sub-skill (see Tables 19 and 20).

In the **questionnaire for the assessment of interactive oral communication skills,** highlight in table 19 the responses provided by the members of the audience – for instance, if the person said "much" for question 5, highlight in the column corresponding to 5 all the "much" answers (see image 7). After highlighting all the answers to all the questions (in table 19), one can establish the result of the assessment done by the audience; the highest level of progress, where the number of responses marked is the highest, will be the current level of development of the competence, while the level just above current level is the zone of proximal development. In the example in image 7, the current level of development of the sub-skill is level 3 (8 of the 11 answers are highlighted) and the zone of proximal development is 4 (where 5 of the 11 answers are highlighted).

Interactive communication (delivered to visitors at a stand, such as in an exhibition/ science fair)		Answers to questions in the assessment questionnaire										
Level	Description of the level	1	2	3	4	5	6	7	8	9	10	11
6	Upon sensing or finding out about the visitors' interests, the student grabs and holds their attention by presenting a captivating and highly relevant part of the research, in a perfectly adjusted language, using the most appropriate tools, engaging in a fluent dialog.	yes	YES	YES	YES	YES	yes	YES	yes	YES	YES	YES
5	Upon sensing the visitors' interests, the student grabs their attention by presenting some of the research, in well-adjusted language, using appropriate tools, which s/he invites the audience to manipulate, play with, etc. He/ she asks and answers questions from the audience.	yes	yes	YES	YES	yes	yes	yes	yes	YES	YES	yes
4	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience by inviting them to manipulate some materials, and addressing simple questions such as: Do you understand the research topic? What are your ideas about ....? What would be your answer to ....?	yes	yes	yes	yes	yes	yes	little/ yes	yes	yes	yes	yes
3	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience to a limited extent. The student uses materials prepared before the interactive communication takes place.	yes	little	little	little	little/ yes	yes	little	yes	little/ yes	little	little
2	The student uses a few materials (paper, computer, etc.) to illustrate some of the points made in the presentation, paying no attention to the visitors' potential interests; fails to engage the visitors.	no	little	no	no	little	no	no	yes	No/ little	no	no
1	Paying no attention to the visitors' potential interest, the student delivers an unadjusted presentation of the research topic, and hands visitors a printed outline, failing to engage them.	no	no	no	no	no/ little	no	no	no	no	no	no

Image 7. Example of a completed table 19

For the **questionnaire for the assessment of oral scientific communication skills**, the procedure is similar. In the example in Image 8, the level of development of the sub-skill is 4 (7 out of 8 answers are highlighted) and the zone of proximal development is at level 5 (5 out of 8 answers are highlighted).

Oral scientific communication (in a plenary)		Answers to questions in the assessment questionnaire							
Level	Description of the level	1	2	3	4	5	6	7	8
6	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student provides clear and relevant answers to all the questions from the audience.	no	yes	YES	YES	YES	yes	yes/ YES	YES
5	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student answers questions from the audience mostly clearly.	no	yes	YES	YES	yes	yes	little/ yes	little/ yes
4	The student delivers an oral presentation of his/ her research work using a clear well-structured visual support (e.g. a slideshow/ PPT). The student answers some questions, but his/ her answers tend to lack clarity.	no	yes	yes	yes	little/ yes	yes	no	no/ little
3	The student delivers an oral presentation of his/ her research work, using visual support (e.g. a slideshow/ PPT). However, the PowerPoint presentation has too much text and/ or confusing visual effects. He/ she makes the presentation sometimes simply reading the text from the slide(s), and refraining from answering questions.	no	yes	little	little	no/ little	no	NA	NA
2	The student delivers a simple oral presentation of his/ her research work, without relying on reading, failing to use visual support to facilitate the understanding of the presentation, and refraining from answering questions.	no	yes	no/ little	no	no	no	NA	NA
1	The student presents his/ her research work orally, by simply reading out his/ her notes in a rather dull manner, and refraining from answering questions.	yes	no	NA	NA	NA	no	NA	NA

Image 8. Example of a completed table 20

3.2.4. Assessment result

For recording the assessment results of the students' oral presentation skills (research results) the teachers use reporting sheets 3 and 4, and for recording the assessment results of the students' writing skills (research results) the teachers use reporting sheets 5 and 6. The manner of using the four sheets is similar to the manner of using reporting sheet 1.

3.2.5. Assessment instruments – interactive oral communication skills for presenting the research results

3.2.5.1. Observation sheet – interactive oral communication

Use the observation sheet immediately after the interactive communication of the research results. Highlight the types of activities and behaviour that the student displayed during the preparation and delivery of the interactive presentation.

Student's full name …… Assessor's name ……

Name of the research topic …….

Name of event ……. Date …….

Interactive communication (delivered to visitors at a stand, such as in an exhibition/ science fair)	
Level	Description of the level
6	Upon sensing or finding out about the visitors' interests, the student grabs and holds their attention by presenting a captivating and highly relevant part of the research, in a perfectly adjusted language, using the most appropriate tools, engaging in a fluent dialog. *For example: involving young students in the analysis of some simple cases, skipping the mathematical proof for an 'unfamiliar with math' audience, pointing out when the audience's questions are similar to his/ her own questions related to the research.*
5	Upon sensing the visitors' interests, the student grabs their attention by presenting some of the research, in well-adjusted language, using appropriate tools, which s/he invites the audience to manipulate, play with, etc. He/ she asks and answers questions from the audience.
4	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience by inviting them to manipulate some materials, and addressing simple questions such as: Do you understand the research topic? What are your ideas about ….? What would be your answer to ….?
3	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience to a limited extent. The student uses materials prepared before the interactive communication takes place. *For example, the student gives examples, provides a schema, plays games with people from the audience, models/ simulates in order to facilitate the audience's understanding of the research topic.*
2	The student uses a few materials (paper, computer, etc.) to illustrate some of the points made in the presentation, paying no attention to the visitors' potential interests; fails to engage the visitors. *For example, the student draws a schema on a sheet of paper, uses a computer for presenting some configurations/ collected data/ etc., uses a board game).*
1	Paying no attention to the visitors' potential interest, the student delivers an unadjusted presentation of the research topic, and hands visitors a printed outline, failing to engage them.

3.2.5.2. Self-assessment sheet – oral interactive communication of the research results

Fill in the self-assessment sheet immediately after you have completed your interactive communication of the research results. Highlight all the types of activities and behaviour you think you demonstrated during the preparation and delivery of the interactive presentation and fill in the second column of the table.

Name of the research topic ……
Event: ……

Interactive communication (oral presentation)	
Student's full name: …… Date of self-assessment: ……	Argue for your statements. Explain what you did specifically and concretely to support your choice.
Upon sensing or finding out about the visitors' interests, I grabbed and held their attention by presenting a captivating and highly relevant part of the research, in a perfectly adjusted language, using the most appropriate tools, engaging in a fluent dialog. *For example: involving young students in the analysis of some simple cases, skipping the mathematical proof for an 'unfamiliar with math' audience, pointing out when the audience's questions are similar to my own questions related to the research.*	
Upon sensing the visitors' interests, I grabbed their attention by presenting some of the research, in well-adjusted language, using appropriate tools, which I invited the audience to manipulate, play with, etc. I asked and answered questions from the audience.	
I provided diverse examples or used various media to present some specific issues related to the research, engaging with the audience by inviting them to manipulate some materials, and addressing simple questions such as: *Do you understand the research topic? What are your ideas about ….? What would be your answer to ….?*	
I provided diverse examples or used various media to present some specific issues related to the research, engaging with the audience to a limited extent. *For example, I provided examples, a schema, played games with people from the audience, modelled/ simulated in order to facilitate the audience's understanding of the research topic. I used materials prepared before the forum.*	
I used a few materials (paper, computer, etc.) to illustrate some of the points made in the presentation, paying no attention to the visitors' potential interests. *For example, I drew a schema on a sheet of paper, used a computer to present some configurations/ data collected / etc., used a board game).*	
I presented the topic of the research providing the same information to all the visitors who stopped by the stand, and perhaps I offered them a sheet of paper containing the general presentation of the results. I failed to engage the audience.	

3.2.5.3. Assessment questionnaire – interactive communication of the research results

For persons from the audience

Name of the student_____

Research topic _____ Data_____

Please tick only one answer to each item.

1. The student identifies the visitors' interests.
   - Yes
   - No
2. The student presents the research findings in a perfectly adjusted language.
   - No
   - Little
   - Yes
   - YES
3. The student grabs the visitors' attention.
   - No
   - Little
   - Yes
   - YES
4. The student holds the visitors' attention.
   - No
   - Little
   - Yes
   - YES
5. The student uses the most appropriate tools/ materials.
   - No
   - Little
   - Yes
   - YES
6. The student asks questions to the audience.
   - Yes
   - No
7. The student asks engaging questions to the audience.
   - No
   - Little
   - Yes
   - YES
8. The student answers questions from the audience.
   - Yes
   - No
9. The student gives appropriate/ relevant answers to questions from the audience.
   - No
   - Little
   - Yes
   - YES
10. The student engages the audience in the research presentation.
    - No
    - Little
    - Yes
    - YES
11. The research presentation is very exciting.
    - No
    - Little
    - Yes
    - YES

Please write your advice for the improvement of the research presentation.

Thank you!

Interactive communication (delivered to visitors at a stand, such as in an exhibition/ science fair)		Answers to questions in the assessment questionnaire										
Level	Description of the level	1	2	3	4	5	6	7	8	9	10	11
6	Upon sensing or finding out about the visitors' interests, the student grabs and holds their attention by presenting a captivating and highly relevant part of the research, in a perfectly adjusted language, using the most appropriate tools, engaging in a fluent dialog.	yes	YES	YES	YES	YES	yes	YES	yes	YES	YES	YES
5	Upon sensing the visitors' interests, the student grabs their attention by presenting some of the research, in well-adjusted language, using appropriate tools, which s/he invites the audience to manipulate, play with, etc. He/ she asks and answers questions from the audience.	yes	yes	YES	YES	yes	yes	yes	yes	YES	YES	yes
4	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience by inviting them to manipulate some materials, and addressing simple questions such as: Do you understand the research topic? What are your ideas about ….? What would be your answer to ….?	yes	yes	yes	yes	yes	yes	Little/ yes	yes	yes	yes	yes
3	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience to a limited extent. The student uses materials prepared before the interactive communication takes place.	yes	little	little	little	little/ yes	yes	little	yes	Little/ yes	little	little
2	The student uses a few materials (paper, computer, etc.) to illustrate some of the points made in the presentation, paying no attention to the visitors' potential interests; fails to engage the visitors.	no	little	no	no	little	no	no	yes	No/ little	no	no
1	Paying no attention to the visitors' potential interest, the student delivers an unadjusted presentation of the research topic, and hands visitors a printed outline, failing to engage them.	no	no	no	no	no/ little	no	no	no	no	no	no

Table 19. Correlation between public's answers collected with the audience questionnaire and the levels of the developmental progression of the interactive communication sub-skills

3.2.6. Assessment instruments – oral scientific communication skills for presenting the research results

3.2.6.1. Observation sheet – oral scientific communication

Use the observation sheet immediately after the communication of the research results. Highlight the types of activities and behaviour that the student displayed during the preparation and delivery of the oral scientific presentation.

Full name of the student …… Assessor's name ……

Research topic …….

Name of event ……. Date …….

Oral scientific communication (in a plenary)	
Level	Description of the level
6	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student provides clear and relevant answers to all the questions from the audience.
5	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student answers questions from the audience mostly clearly.
4	The student delivers an oral presentation of his/ her research work using a clear well-structured visual support (e.g. a slideshow/ PPT). The student answers some questions, but his/ her answers tend to lack clarity.
3	The student delivers an oral presentation of his/ her research work, using visual support (e.g. a slideshow/ PPT). However, the PowerPoint presentation has too much text and/ or confusing visual effects. He/ she makes the presentation sometimes simply reading the text from the slide(s), and refraining from answering questions.
2	The student delivers a simple oral presentation of his/ her research work, without relying on reading, failing to use visual support to facilitate the understanding of the presentation, and refraining from answering questions.
1	The student presents his/ her research work orally, by simply reading out his/ her notes in a rather dull manner, and refraining from answering questions.

3.2.6.2. Self-assessment sheet – oral scientific communication of the research results

Fill in the self-assessment sheet immediately after you have completed your presentation in the congress/ symposium. Highlight all the types of activities and behaviour you think you demonstrated during the preparation and delivery of the oral scientific presentation and fill in the second column of the table.

Name of the research topic ……
Event: ……

Oral scientific communication (in conferences/ congresses/ scientific symposia)	
Full name of the student: …… Date of self-assessment: ……	Argue for your statements. Explain what you did specifically and concretely to support your choice.
I have delivered an oral presentation of my research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). I provided clear and relevant answers to all the questions from the audience.	
I have delivered an oral presentation of my research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). I answered questions from the audience mostly clearly.	
I have delivered an oral presentation of my research work using a clear well-structured visual support (e.g. a slideshow/ PPT). I answered some questions, but they tended to lack clarity.	
I have delivered an oral presentation of my research work, using visual support (e.g. a slideshow/ PPT). However, the PowerPoint presentation I prepared has too much text and/ or confusing visual effects. I made the presentation sometimes simply reading the text from the slide(s), and refrained from answering questions from the audience.	
I have delivered a simple oral presentation of my research work. I sometimes read some of the information from a sheet, and the visual support I used could facilitate the understanding of my presentation if improved. I refrained from answering questions from the audience.	
I have made an oral presentation by reading information about my research work from my notes/ sheet, without using visual support. I refrained from answering questions from the audience.	

3.2.6.3. Assessment questionnaire - oral scientific communication of the research results

For people from the audience

Name_____

Research topic _____ Date_____

Please tick only one answer to each item.

1. The student presents his/ her research work orally, by simply reading the presentation.

   ☐ Yes ☐ No

2. The student uses visual support (e.g. slideshow/ PPT).

   ☐ Yes ☐ No

3. The visual support (e.g. PPT/ slideshow) used in the presentation is clear.

   ☐ No ☐ Little ☐ Yes ☐ YES ☐ NA (there is no visual support)

4. The visual support (e.g. PPT/ slideshow) used in the presentation is highly structured.

   ☐ No ☐ Little ☐ Yes ☐ YES ☐ NA (there is no visual support)

5. The visual support (e.g. PPT/ slideshow) used in the presentation is a professional one (design, word density, slides density, slides' effects, etc.)

   ☐ No ☐ Little ☐ Yes ☐ YES ☐ NA (there is no visual support)

6. The student answers questions from the audience.

   ☐ Yes ☐ No

7. The student provides clear answers to all the questions from the audience.

   ☐ No ☐ Little ☐ Yes ☐ YES ☐ NA (the student gives no answer)

8. The student provides relevant answers to all the questions from the audience.

   ☐ No ☐ Little ☐ Yes ☐ YES ☐ NA (the student gives no answer)

Please write your advice for the improvement of the research presentation.

Thank you!

Oral scientific communication (in a plenary)		Answers to questions in the assessment questionnaire							
Level	Description of the level	1	2	3	4	5	6	7	8
6	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student provides clear and relevant answers to all the questions from the audience.	no	yes	YES	YES	YES	yes	yes/ YES	YES
5	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student answers questions from the audience mostly clearly.	no	yes	YES	YES	yes	yes	Little/ yes	little/ yes
4	The student delivers an oral presentation of his/ her research work using a clear well-structured visual support (e.g. a slideshow/ PPT). The student answers some questions, but his/ her answers tend to lack clarity.	no	yes	yes	yes	little/ yes	yes	no	no/ little
3	The student delivers an oral presentation of his/ her research work, using visual support (e.g. a slideshow/ PPT). However, the PowerPoint presentation has too much text and/ or confusing visual effects. He/ she makes the presentation sometimes simply reading the text from the slide(s), and refraining from answering questions.	no	yes	little	little	no/ little	no	NA	NA
2	The student delivers a simple oral presentation of his/ her research work, without relying on reading, failing to use visual support to facilitate the understanding of the presentation, and refraining from answering questions.	no	yes	no/ little	no	no	no	NA	NA
1	The student presents his/ her research work orally, by simply reading out his/ her notes in a rather dull manner, and refraining from answering questions.	yes	no	NA	NA	NA	no	NA	NA

Table 20. Correlation between public's answers collected with the assessment questionnaire and the levels of the developmental progression of the oral scientific communication subskills

3.2.7. Assessment instruments – written scientific communication skills for presenting the research results

3.2.7.1. Observation sheet – written scientific communication

Use the observation sheet immediately after the group of students has finished writing the article. Highlight the types of activities and behaviour that the group members have displayed during the preparation of the article; keep in mind the final version of the article.

Name ……

Research topic ……. Date …….

Written scientific communication (research article)	
Level	Description of the level
6	Seamlessly integrating each members' contribution, the group of students has produced a well-structured, well written and neatly typed research article, which presents the research topic, the approach, the results & the mathematical proof(s) in a highly appropriate language.
5	Incorporating the contribution of each member of the team, the group of students has produced a well-structured, neatly typed research article which presents the research topic, the approach, the results & the mathematical proof(s), in need of limited editing.
4	Some members of the group of students have contributed to producing a neatly typed, quite well-structured and mostly appropriately worded research article, which contains the research topic, their research approach, the research results & the mathematical proof(s).
3	Some members of the group of students or a member of the group has produced a complete although poorly structured research article, typed in a rather messy manner, in a register that fails to comply with the standards of a research article.
2	Some members of the group of students or a member of the group has typed up an incomplete and poorly structured article, which presents the topic and the research that was carried out, but fails to present the scientific results, etc.
1	Some members of the group of students or a member of the group has typed up a simplistic, partial report on the research they have carried out, failing to specify essential information e.g. the topic and/or the approach and/ or results, etc.

3.2.7.2. Self-assessment sheet – written scientific communication of the research results

Fill in the self-assessment sheet immediately after you have completed the final form of the research article. When you fill in the form, all students in your group must be involved. After you discuss together the statements in the table, you highlight the types of activities and behaviour you think you demonstrated during the process of producing the article and fill in the second column of the table.

Full name of the students in the group ……

……

Written scientific communication (research article)	
Name of the research topic: …… Date of self-assessment: ……	Argue for your statements. Explain what you did specifically and concretely to support your choice.
We have all contributed to the writing of the research article, which is well structured, well written and neatly typed, which presents the research topic, the approach, the results & the mathematical proof(s).	
We have all contributed to the writing of the research article, which is well structured, which presents the research topic, the approach, the results & the mathematical proof(s), which is neatly types – it only needs small corrections (formulations and/ or typing).	
Some of us have contributed to the writing of the research article, which is correctly written (we used texts written by each, we inserted mathematical formulae, etc.), quite well structured, in an acceptable language, which presents the research topic, the approach, the results & the mathematical proof(s).	
Some (or one) of us typed on a computer a complete research article, which is poorly structured, with an untidy look, which does not respect the standards of writing a scientific article.	
Some (or one) of us typed on a computer an incomplete poorly structured research article, which shares the research topic and the research we carried out, but does not share the research results etc.	
Some (or one) of us typed on a computer a partial report on the research we carried out, without mentioning essential information – e.g. the research topic and / or the approach and / or the results, etc.	

3.2.8. Assessment instruments – synthetic and visual communication skills for presenting the research results

3.2.8.1. Observation sheet – synthetic visual communication

Use the observation sheet immediately after the group of students has finished developing the poster. Highlight the types of activities and behaviour that the group members have displayed during the preparation of the poster; keep in mind the final version of the poster.

Name ……

Research topic ……. Date …….

Synthetic visual communication (poster)	
Level	Description of the level
6	The group has produced and made available online a very clear and attractive item of visual communication (poster in pdf format, very well edited video recording), which shares the research topic, as well as all the major results in an original manner, using highly appropriate computer software/ tools to thoroughly engage viewers.
5	The group has produced a very clear and attractive item of visual communication which shares the research topic as well as all the major results, using computer software/ tools to show the most relevant parts of it.
4	The group has produced a clear and attractive item of visual communication which shares the research topic as well as most of the results using computer software/ tools to show some parts of it.
3	The group has produced an attractive although not thoroughly clear item of visual communication of the research topic and some results using computer software/ tools to show some parts of it.
2	The group has produced a simple item of visual communication of the research topic and some results, which however tends to be mostly confusing and/or overloaded, blurring the key points to get across.
1	The group of students has produced a simplistic item of visual communication of the research topic (a poster), which fails to attract viewers' interest.

3.2.8.2. Self-assessment sheet – synthetic and visual communication of the research results

Fill in the self-assessment form immediately after you have completed the final form of the scientific poster (sharing your research). When you fill in the form, all students in your group must be involved. After you discuss together the statements in the table, you highlight the types of activities and behaviour you think you demonstrated during the process of producing the poster and fill in the second column of the table.

Full name of the students in the group ……..

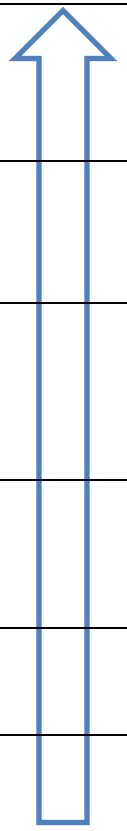
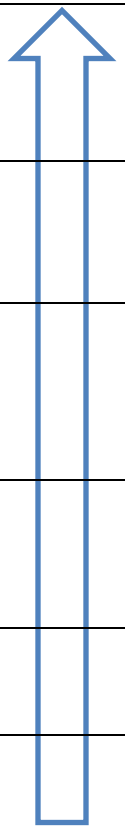
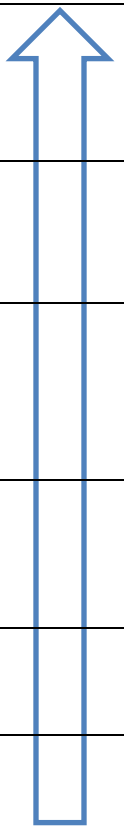
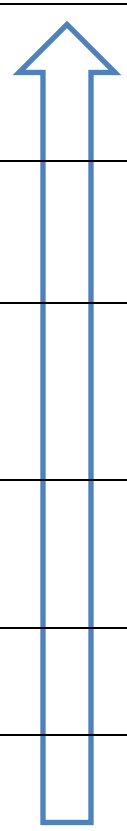
……

Synthetic visual communication (poster)	
Name of the research topic: …… Date of self-assessment: ….....	Argue for your statements. Explain what you did specifically and concretely to support your choice.
We have produced together, using various IT programmes, a very attractive and clear poster (available online), which shares the research topic and the major research results in an original manner.	
We have produced together, using various IT programmes, a very attractive and clear poster, which shares the research topic and the major research results.	
We have produced together, using IT instruments, an attractive and clear poster, which shares the research topic and most of the research results.	
We have produced together, using IT instruments, an attractive poster which is not clear enough, which shares the research topic and some of the research results.	
We have produced together, without using IT, a confusing and messy or overloaded poster, which shares the research topic and a few research results.	
We have produced together, without using IT, a simple unattractive poster, which shares the research topic.	

3.2.9. Reporting sheets

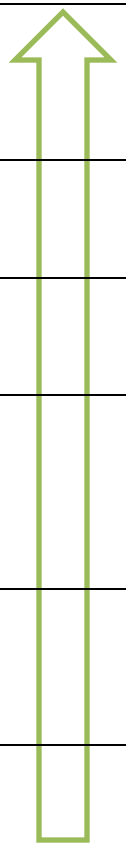
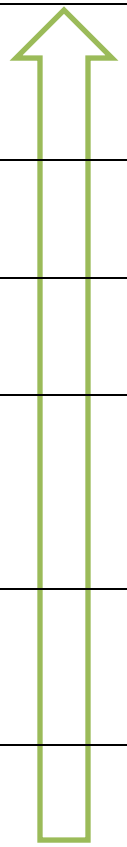
Reporting sheet 3

Name: _____ Date: _____

Interactive communication (delivered to visitors at a stand, such as in an exhibition/ science fair)					
Level	Description of the level	**Teacher assessment**	**Student assessment**	**Audience Assessment**	**Peer Assessment**
6	Upon sensing or finding out about the visitors' interests, the student grabs and holds their attention by presenting a captivating and highly relevant part of the research, in a perfectly adjusted language, using the most appropriate tools, engaging in a fluent dialog.				
5	Upon sensing the visitors' interests, the student grabs their attention by presenting some of the research, in well-adjusted language, using appropriate tools, which s/he invites the audience to manipulate, play with, etc. He/ she asks and answers questions from the audience.				
4	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience by inviting them to manipulate some materials, and addressing simple questions such as: Do you understand the research topic? What are your ideas about ….? What would be your answer to ….?				
3	The student provides diverse examples or uses various media to present some specific issues related to the research, engaging with the audience to a limited extent. The student uses materials prepared before the interactive communication takes place.				
2	The student uses a few materials (paper, computer, etc.) to illustrate some of the points made in the presentation, paying no attention to the visitors' potential interests; fails to engage the visitors.				
1	Paying no attention to the visitors' potential interest, the student delivers an unadjusted presentation of the research topic, and hands visitors a printed outline, failing to engage them.				

Reporting sheet 4

Name: _____ Date: _____

Oral scientific communication (in a plenary)					
Level	Description of the level	**Teacher assessment**	**Student assessment**	**Audience Assessment**	**Peer Assessment**
6	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student provides clear and relevant answers to all the questions from the audience.				
5	The student delivers an oral presentation of his/ her research work using a clear and highly structured visual support (e.g. a slideshow/ PPT). The student answers questions from the audience mostly clearly.				
4	The student delivers an oral presentation of his/ her research work using a clear well-structured visual support (e.g. a slideshow/ PPT). The student answers some questions, but his/ her answers tend to lack clarity.				
3	The student delivers an oral presentation of his/ her research work, using visual support (e.g. a slideshow/ PPT). However, the PowerPoint presentation has too much text and/ or confusing visual effects. He/ she makes the presentation sometimes simply reading the text from the slide(s), and refraining from answering questions.				
2	The student delivers a simple oral presentation of his/ her research work, without relying on reading, failing to use visual support to facilitate the understanding of the presentation, and refraining from answering questions.				
1	The student presents his/ her research work orally, by simply reading out his/ her notes in a rather dull manner, and refraining from answering questions.				

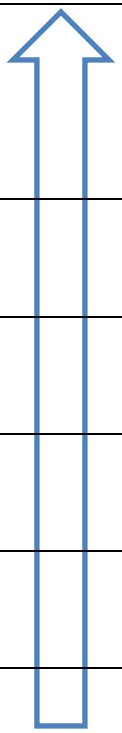
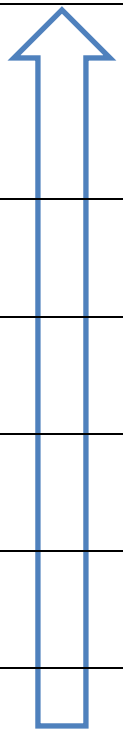
Reporting sheet 5

Name: _____ Date: _____

Written scientific communication (research article)			
Level	Description of the level	**Teacher assessment**	**Student assessment**
6	Seamlessly integrating each members' contribution, the group of students has produced a well-structured, well written and neatly typed research article, which presents the research topic, the approach, the results & the mathematical proof(s) in a highly appropriate language.		
5	Incorporating the contribution of each member of the team, the group of students has produced a well-structured, neatly typed research article which presents the research topic, the approach, the results & the mathematical proof(s), in need of limited editing.		
4	Some members of the group of students have contributed to producing a neatly typed, quite well-structured and mostly appropriately worded research article, which contains the research topic, their research approach, the research results & the mathematical proof(s).		
3	Some members of the group of students or a member of the group has produced a complete although poorly structured research article, typed in a rather messy manner, in a register that fails to comply with the standards of a research article.		
2	Some members of the group of students or a member of the group has typed up an incomplete and poorly structured article, which presents the topic and the research that was carried out, but fails to present the scientific results, etc.		
1	Some members of the group of students or a member of the group has typed up a simplistic, partial report on the research they have carried out, failing to specify essential information e.g. the topic and/or the approach and/ or results, etc.		

Reporting sheet 6

Name: _____ Date: _____

Visual communication (poster)			
Level	Description of the level	**Teacher assessment**	**Student assessment**
6	The group has produced and made available online a very clear and attractive item of visual communication (poster in pdf format, very well edited video recording), which shares the research topic, as well as all the major results in an original manner, using highly appropriate computer software/ tools to thoroughly engage viewers. .		
5	The group has produced a very clear and attractive item of visual communication which shares the research topic as well as all the major results, using computer software/ tools to show the most relevant parts of it.		
4	The group has produced a clear and attractive item of visual communication which shares the research topic as well as most of the results using computer software/ tools to show some parts of it.		
3	The group has produced an attractive although not thoroughly clear item of visual communication of the research topic and some results using computer software/ tools to show some parts of it.		
2	The group has produced a simple item of visual communication of the research topic and some results, which however tends to be mostly confusing and/or overloaded, blurring the key points to get across.		
1	The group of students has produced a simplistic item of visual communication of the research topic (a poster), which fails to attract viewers' interest.		

3.3. Methods and instruments for assessing the students' competence in *making use of aids and tools (IT included)*

For assessing students' competence in *making use of aids and tools (MUAT)* developed within the MeJ workshops, we suggest the following methods and instruments:

Methods	Instruments
Observation Analysis of the information and reflections in the student's logbooks (twice per school year) Analysis of the student's articles/ oral presentation (once per school year) Analysis of the student's reporting sheet (MUAT) (twice per school year)	The MUAT observation sheet

Table 21. Methods and instruments for assessing the students' competence in *making use of aids and tools*

The students' logbook, articles, oral presentations, and the student reporting sheets (MUAT) are useful only for identifying aspects mentioned in the observation sheet which the teacher didn't/ couldn't observe during the MeJ weekly meetings or for checking the accuracy of the observed actions and behaviour. If there is no concordance between the teacher's observations and the student logbook/ article/ oral presentation/ reporting sheet, the teacher might decide to take a closer look at the specific discordant issues and to have a short interview with the student.

3.3.1 How to use the observation sheet?

Filling in the observation sheet

In order to fill in the observation sheet, the teacher observes continuously and notes the student's activities and behaviour, and analyses the documents and products mentioned above.

The MUAT reporting sheet, filled in by the student, provides important information for filling in the observation sheet – especially information relative to the student's capacity to use tools and aids reflexively. The student fills in the sheet throughout the year. This does not mean that the student needs to fill in the form in every activity of the research workshop; as the use of aids and tools may involve even developing an IT programme, it is very unlikely that the student can finish such a programme during the time of a weekly meeting (1 hour). We suggest that the students make entries in the research diary about the use of aids and tools (so that they remember important aspects) and, once every three months, for instance, they fill in the MUAT reporting sheet. We share below the student reporting sheet (MUAT).

The student reporting sheet (MUAT)

Name: …… Date: ……..

Please fill in this sheet for each tool or aid you use while working on the research topic in the MeJ workshop. Please be as specific as possible.

Name of the tool/ aid	
I know about this tool from ….	
The possible uses of this tool/ aid are …	
The limitations of this tool/ aid are ….	
I used this tool because …	
I used this tool for …	
Using this tool, I managed to …	
Using this tool, I learnt …	

Please feel free to add any comment and/ or photo related to the use of the tool/ aid.

After filling it in, please send this sheet to your MeJ teacher/ facilitator (by email).

The teacher fills in the MUAT observation sheet twice during the academic year. We recommend that the periods for filling in the observation sheets should be January and May. Before completing the sheet, the teacher asks the students to give her a copy of their (filled in) MUAT reporting sheet.

Remember!

The MUAT observation sheet and the student reporting sheet have to be discussed with the MeJ students at the beginning of the school year. They have to understand what we expect from them – that is why modelling actions and behaviour or provision of examples thereof are a must. The teacher should also guide the students on how to fill the students report sheet.

In order to fill in the observation sheet, the teacher does the same as in the case of filling in the CPS observation sheet (see subchapter 3.1.1.), in other words, she marks with a highlighter/ colour the activities and the behaviour observed at the student.

Interpreting the information in the observation sheet (completed)

The highest level of the developmental progression where all (or almost all) the activities or behaviour are marked will be considered the student's level in the development of the collaborative problem solving competence, while the level on the progression where about half the activities or behaviour are marked is the student's zone of proximal development.

3.3.2. Assessment result

In order to record the result of assessing the competence *making use of aids and tools*, we use Reporting sheet 7. The manner of work is similar to that described in subchapter 3.1.3, the difference being that in the case of this competence we only have the result of the teacher-conducted assessment.

Comparison between assessments at the beginning of January and those at the end of May will inform the development of each student's competence in *making use of aids and tools* within the MeJ workshop. It is likely that in the case of some students, there will be no difference between the two assessments because in the second half of the academic year the students have found solutions to the research problem, and their activity is more focused on communicating their results.

If the student attends the MeJ workshop two (or more) consecutive years, it makes sense to compare results of this assessment across the two (or more) years. We assume that as long as one is doing the math research activity, there can normally be stagnation or growth of the competency.

3.3.3. The MUAT observation sheet

Use the observation sheet at the beginning of January and at the end of May of each school year. Mark with highlighters (use different colours for the two dates) to identify the kinds of activities and behaviours that the student demonstrates.

Level	**Student's name:**	**Date 1:** **Date 2:**
6	The student identifies and uses familiar or non-familiar, complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures for determining mathematical solutions. Constructs new tool(s) - necessary for understanding and solving the research task. Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool. Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used, and why s/he ended up not using those aids and tools.	
5	The student identifies and uses familiar or non-familiar, complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures for determining mathematical solutions. Has the idea to create a new tool or aid necessary for understanding and solving the research task (e.g. a computer application). Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool. Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used.	
4	The student identifies and uses familiar or non-familiar aids and tools for implementing processes and procedures for determining mathematical solutions. He learns how to use the non-familiar aids and tools. Using the tools involves a sequence of process or linking different information. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means. Evaluates the use of the aids and tools and communicates their learning from this experience.	
3	The student identifies and uses familiar aids and tools. Using the tool involves a sequence of process or linking different information. The aids and tools used may assist in implementing processes and procedures to determine mathematical solutions. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means. Evaluates the use of the aids and tools without sharing their learning from this experience.	
2	The student identifies and uses tools and aids s/he already used during math lessons for formulating the research task mathematically. Describes, in general terms, the possibilities and the limitations of the used tools and aids. Analyses the use of both aids and tools (which aids and tools they have used, why they used these aids and tools).	
1	The student identifies and uses familiar tools and aids (e.g. measuring instruments, objects that are mentioned in the text of the research task). S/he describes, in general terms, the possibilities and the limitations of the used tools. S/he uses the tools and aids in order to recognise mathematical structures in the text of the research topic. S/he names the used tools and formulates one reason for using the identified tools.	

3.3.4. Reporting sheet

Reporting sheet 7

Student's name: _____ Date: _____

Level	Level description	Teacher assessment
6	The student identifies and uses familiar or non-familiar, complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures for determining mathematical solutions. Constructs new tool(s) - necessary for understanding and solving the research task. Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool. Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used, and why s/he ended up not using those aids and tools.	
5	The student identifies and uses familiar or non-familiar, complex aids and tools (with multiple functionalities) which may assist in implementing processes and procedures for determining mathematical solutions. Has the idea to create a new tool or aid necessary for understanding and solving the research task (e.g. a computer application). Interprets the resulting findings from using the tools and aids. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means, after reflecting to understand and evaluate the merits and limitations of the tool. Evaluates the use of the aids and tools and communicates their learning from this experience. Reflects on what else s/he could have used.	
4	The student identifies and uses familiar or non-familiar aids and tools for implementing processes and procedures for determining mathematical solutions. He learns how to use the non-familiar aids and tools. Using the tools involves a sequence of process or linking different information. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means. Evaluates the use of the aids and tools and communicates their learning from this experience.	
3	The student identifies and uses familiar aids and tools. Using the tool involves a sequence of process or linking different information. The aids and tools used may assist in implementing processes and procedures to determine mathematical solutions. Describes the possibilities and the limitations of the used aids and tools by illustrating what s/he means. Evaluates the use of the aids and tools without sharing their learning from this experience.	
2	The student identifies and uses tools and aids s/he already used during math lessons for formulating the research task mathematically. Describes, in general terms, the possibilities and the limitations of the used tools and aids. Analyses the use of both aids and tools (which aids and tools they have used, why they used these aids and tools).	
1	The student identifies and uses familiar tools and aids (e.g. measuring instruments, objects that are mentioned in the text of the research task). S/he describes, in general terms, the possibilities and the limitations of the used tools. S/he uses the tools and aids in order to recognise mathematical structures in the text of the research topic. S/he names the used tools and formulates one reason for using the identified tools.	

4. Conclusions

The assessment methodologies of the three competences were tested in the 2015-2016 academic year, in the math research workshops, by three math teachers and 51 students from the Lycée d'Altitude de Briançon (France) and 4 math teachers and 39 students from the Colegiul Național Emil Racoviță Cluj-Napoca (Romania).

In order to document the testing of the assessment methodologies, each teacher wrote in their own *Logbook* aspects related to the use of the assessment methods and instruments, reflections on their usefulness, complexity, relevance, proposals for improving the assessment instruments and the assessment methodologies; in Chapter 3 of this guidebook we included the revised version of the three assessment methodologies.

Image 9. Analysis of the findings from testing the competence assessment methodologies

Below, we are sharing some aspects that we find worth remembering from the analysis of the testing phase of the assessment methodologies.

It was worth the effort!

- the 7 teachers believe that the assessment instruments are very useful and relevant;
- 5 of the 7 teachers consider that the instruments are easy to use;

- The explanation and use of the self-assessment sheets by the students have contributed to the better understanding of the activities included in the research workshops (especially for the students who had not taken part in such workshops before);
- use of the assessment methodologies allowed us to document the students' progress in developing the three competences; "my effort was rewarded when I compared the reporting sheets from December to those in May and I actually saw the students' progress"; in fact, although we started with the idea that competence assessment in this case is formative, we found that the instruments can also be used for summative purposes, by reporting the results obtained at the end of the school year.

Experience matters!

- The two teachers who had not had previous experience in using observation sheets (not in conducting formative assessment) considered that the use of the assessment instruments was difficult – "it was difficult to use the assessment instruments because I had not used an observation sheet before and I am not familiar with this type of assessment";
- "The difficulties we encountered were due to the fact that the students are not used to doing self-assessment and to competence assessment".

Nothing is perfect!

- "Many sheets to fill in and a lot of time needed for filling in sheets".

To keep in mind for next year!

- "The self-assessment sheets and the reporting sheet should be included in each student's Research diary; the students should have them in both electronic and printed form."
- "The self-assessment sheets should be displayed in the classroom where the research workshop is taking place. It is useful to see the progression in the competence development, for both teachers and students."
- "The teacher should make a note of what s/he observes during the research workshops (a key term and the student's name) – otherwise it is difficult to remember, 2-3 months later, what the student did. I myself, starting in January, made brief notes of what I observed, at the end of each activity – this helped a lot."
- "We started discussions about the contents of the assessment instruments in a group activity. In each group I included 1-2 students who had participated in the research workshop in the previous year; they got engaged in the discussions and provided numerous examples. After the small group discussion, I organised a full group discussion with all the students, and we focused on the unclear aspects and checked, randomly, the understanding of certain behaviour that occurs in the sheet. This manner of organising the discussions about the assessment sheet was very effective."
- In an academic year, it is enough to assess the development of 1-2 sub-skills of the oral and written communication skills.

- "We discussed with the students the self-assessment of the sub-competence *interactive oral communication*, two weeks before the Research project Forum in our school, during the preparation of the presentations. The students conceived of the self-assessment sheet as a guidance for preparing the presentation. I will do the same next year!"
- "Some students completed a MUAT reporting sheet for each tool/ aid, others had one sheet only, which they kept revising. We can allow students to work as they feel better – it does not matter how many reporting sheets they complete, it matters that they reflect on the use of aids and tools."
- The better organised and prepared the assessment process is, the shorter the time needed for assessment.

The MatLan project provided the opportunity to initiate the assessment of competences developed by students in the math research workshops. The collaboration among math teachers from the two schools, as well as the support received by the experts of *MATh.en.JEANS Association*, *ANIMATH Association*, and the *Romanian Reading and Writing for Critical Thinking Association* have been essential. This has been an ambitious approach in the context in which specialists in education are looking for solutions to put into practice the assessment of key competences in European Union countries. One of the main messages by KeyCoNet shows that policy-makers and practitioners need to pay attention to the assessment of key competences, and that in this respect efforts need to be made to pilot and evaluate various assessment methods (Grayson, 2014).

The assessment methodologies of the three competences may certainly be improved. Moreover, there should be assessment methodologies developed for the other competences that students develop in the research workshops (see Figure 2.) Our approach in the development of the assessment methodology (see subchapter 1.3. Our approach to the development of the assessment methodology) can be used for the development of assessment methodologies of the other competences developed in the research workshop as well, and – why not – for the development of assessment instruments of competences developed in any school subject.

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Index

The guidebook addresses teachers and experts in didactics who are interested in developing competence assessment tools. We are confident that the process of developing the competence assessment methodology / instruments described in this guidebook may prove useful for specialists interested in competence assessment.

The guidebook is structured in four chapters. The first chapter presents aspects connected to assessment in the mathematics research workshops – what we know about how assessment is done in such research workshops, why we aim to assess competences students develop in the research workshops, and some aspects that should be kept in mind when assessing competences. In the second chapter, we share the diagram of competences students develop in the research workshops, and operationalise / define the three competences students develop in these workshops: *collaborative problem solving*, *use of aids and tools,* and *written and oral communication skills for sharing the research results.* Chapter three includes the methodology of assessing the above-mentioned competences, which was tested over the period of an academic year, and then revised. The last chapter shares the conclusions we drew upon testing the assessment methods and tools, as well as a few ideas related to how our approach can be continued.

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