

a) Let n be a natural number. Numbering twice the number of functions $f : \{1, 2, 3, \dots, n\} \rightarrow \{0, 1\}$ prove that $C_n^0 + C_n^1 + \dots + C_n^n = 2^n$.

b) Starting from this, can we prove that $(a + b)^n = C_n^0 a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \dots + C_n^n b^n$, for any $a, b \in \mathbb{R}$?